INTERNATIONAL TRANSMISSION OF MONETARY SHOCKS WITH INTEREST RATE RULE

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Abstract

This paper explores the implications of monetary policy rules in the general equilibrium two-country framework of Obstfeld and Rogo¤ (1995). It is argued that the sign of the correlation of domestic and foreign outputs can be positive after a monetary shock, contrary to the standard result. The reason is that an interest rate rule targeting the consumer price index implies less volatile terms of trade and this reduces the expenditure switching effect, and thus the demand effect through the fall of the real interest rate prevails. It is also shown that inertia in the interest rate rule is a necessary condition for the model to display persistence of the real variables after a shock to the interest rate rule.

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1 Introduction

The analytical tools for studying the international transmission of monetary shocks were developed in the 1960s by Robert Mundell (1962, 1968) and Marcus Fleming (1962), and the resulting Mundell-Fleming model has been the workhorse of both theorists and policy-makers for the last decades. The two-country version of the model predicts that a monetary expansion in one country will produce an increase in home output and a negative output response in the other country. The transmission mechanism is the trade balance: the home country will lower its interest rate, depreciate the exchange rate and gain competitiveness, thus producing an expenditure switching effect from foreign goods to home goods, resulting in an improvement of the home country trade balance, an increase in its output and a decrease in foreign output, the so-called beggar-thy-neighbour effect. On the financial side of the model, perfect capital mobility requires interest rates to equal (in the version with static expectations), hence foreign interest rate will decrease and this should make foreign output grow. However, without an increase in foreign real money balances, the positive slope of the LM curve implies that a lower interest rate can only be matched with lower output.

The equations of the Mundell-Fleming model were ad-hoc, in the sense that they were not the result of the optimizing behaviour of agents. In the mid-80s Keynesian economists began to build a framework in which the optimizing behaviour of both firms and consumers could account for the empirical short-run non-neutrality of money. Imperfect competition is the framework that allows the firms to fix prices and to introduce some reason for these prices not to be changed in the face of small variations in demand. This reason is the cost of adjustment of prices or "menu costs". The resulting model is sometimes referred to as the "New Neoclassical Synthesis", as in Goodfriend and King (1997).

In the field of international macroeconomics, this setup was not implemented until recently, with the seminal Obstfeld and Rogoys (1995) model (hereafter OR). The OR or redux model is a simple two country environment, lacking some fundamental features such as stochastic shocks, investment and capital accumulation, price dynamics, etc. The purpose of the model was to show that nominal shocks can have real effects in both countries in the short run and even in the long run in an optimizing general equilibrium model, provided some nominal price rigidity is in place. Some of their results were surprising: monetary shocks can have long lasting real effects on both countries due to the wealth reallocation that produces the short run current account imbalance. Another important feature of this framework is that it is suitable for welfare analysis. The traditional Keynesian model
stressed the beggar-thy-neighbour result of a monetary expansion that leads to a depreciation and to an expenditure switching effect. However, Obstfeld and Rogo‡ proved that in their model this effect is only of second order on utility on both countries, while there is a rst order positive effect due to the increase in aggregate world demand and output, resulting in an equal welfare improvement for both countries. These features have been so fruitful that a large literature has developed under the title of New Open Economy Macroeconomics.\footnote{There is a useful web page about this research program where contributions are listed and, in most cases, available to download: www.princeton.edu/~bmdoyle/open.html.} In this strand progress has been made during the last few years in a number of directions: some extensions have explored the relevance of critical parameters, the stochastic environment has been analytically fully†edged (Obstfeld and Rogo‡ 1998 and 2000), the model has been adapted to the pricing to market (PTM) behaviour of rns and staggered price setting has been introduced, along with capital accumulation to provide a complete business cycle model. Lane (1999) provides a comprehensive survey. One of the remaining challenges is to build dynamic stochastic general equilibrium models with powerful persistent mechanisms to account for lasting effects of money on output and with powerful international transmission mechanisms to explain the positive and signi…cant comovement of output.

The OR model adds another transmission mechanism: the link between real interest rates and, through the consumers intertemporal behaviour, between consumption in both countries. The fall of home real interest rate brings about the fall of the foreign one, and the consequence is that both aggregate consumptions increase. However, this intertemporal effect is shadowed by the expenditure switching effect, resulting in the same negative correlation between home and foreign outputs, as in the MF model. Moreover, as foreign consumption increases there is a negative comovement of consumption and output, which is clearly at odds with the strongly procyclical behaviour of consumption found in business cycle research. Recent empirical research using vector autorregresive (VAR) models has found a different story: both home and foreign output increase. Betts and Devereux (2000) and Kim (1999) nd a positive response of foreign outputs to US expansionary monetary shocks.\footnote{There are other VAR models estimated to address the effects of monetary shocks, but they are generally concerned only with the movement of the real exchange rate and the current account. The general conclusion is that the real exchange rate depreciates and the current account improves, as suggested in the MF and OR models. Recent contributions include Lane (2000), Prasad (1999) and Rogers (1999).} The latter is a detailed empirical contribution that concludes that the trade balance is not the main international transmission
channel (as in Canova and Dellas, 1993) because the magnitude of the trade changes is small compared to output changes and because the direction of change of the trade balance and the output is not always the predicted one. Kim also finds evidence that a US monetary expansion decreases real interest rates both at home and abroad and that this causes an increase in consumption and investment at home and abroad, thus rising aggregate demand and output, in both countries. Hence, the intertemporal channel seems to be the one which prevails.

In this paper I argue that if both countries follow a monetary rule targeting the consumer price index (which includes import prices and thus the effect of nominal exchange rates) then, when the home country lowers its nominal interest rate, the other will do the same to prevent an increase in the consumer price index (CPI). This response implies a positive correlation of nominal interest rates that automatically stabilizes the nominal exchange rate, thus reducing the expenditure switching effect and predicting, for reasonable parameter values, a positive comovement of home and foreign outputs. The second claim of the paper is that the inertia of the interest rule is necessary to generate persistent real effects of monetary shocks, as measured by the autocorrelation of home output. Without this inertia in the interest rule there is no real persistence even with staggered prices a la Calvo. The reason is that the one-shot, transitory variation of the interest rate brings about a one-shot, transitory variation of the exchange rate, thus disturbing the CPI in just one period. If the share of imports on the consumption basket is important, then staggered home prices are not enough to display inertia in the CPI: inertia in the nominal exchange rate is also needed, and this is achieved by the inertia in the interest rule.

The structure of the paper is as follows. The next section describes the model, section 3 explains the monetary policy rule, and in section 4 a nominal persistence mechanism is incorporated to the model via staggered prices a la Calvo and inertia in the interest rule. Section 5 provides the concluding comments.

2 The model

The simplest general equilibrium model in which we can explore the implications of monetary policy rules is the OR redux model, with the modification developed by Tille (2000) to distinguish between cross-country and within-country elasticities of substitution.
2.1 Setup

There is a continuum of individuals, \( z \in [0;1] \), distributed in two countries: home, with \( z \in [0;n] \), and foreign with \( z \in (n;1] \). Each individual is a consumer and a producer, and has the utility function:

\[
U_t = \frac{1}{s^t} u_s(C; \frac{M}{P}; Y(z))
\]

\[
= \frac{1}{s^t} \frac{1}{2} \log C_s + \bar{A} \log P_s i \frac{1}{2} Y_s(z)^2
\]

(1)

where \( 0 < \bar{\theta} < 1; \bar{A} > 0 \). The definition of the consumption index \( C \) allows for discrimination between home and foreign goods:

\[
C = n^\frac{1}{\bar{\theta}} C^h + (1 - n)^\frac{1}{\bar{\theta}} C^f
\]

where \( C^h; C^f \) are the subindexes of the consumption of home produced goods and foreign produced goods respectively, and \( \bar{\theta} \) is the elasticity of substitution between them (the cross-country elasticity) that we will assume greater than one throughout the paper. The consumption subindexes are:

\[
C^h = n^\frac{1}{\bar{\theta}} \int_{0}^{1} \frac{1}{n} \frac{C^h(z)}{P^h(z)} \frac{1}{\mu} \frac{1}{2} \frac{1}{n} \mu \, dz
\]

\[
C^f = (1 - n)^\frac{1}{\bar{\theta}} \int_{n}^{1} \frac{1}{n} \frac{C^f(z)}{P^f(z)} \frac{1}{\mu} \frac{1}{2} \frac{1}{n} \mu \, dz
\]

where \( \mu \) is the elasticity of substitution between goods of the same country (the within-country elasticity). The cost minimizing price indexes are:

\[
P^h = n^\frac{1}{\bar{\theta}} \int_{0}^{1} \frac{1}{n} \frac{1}{n} \frac{P^h(z)}{C^h(z)} \frac{1}{\mu} \frac{1}{2} \frac{1}{n} \mu \, dz
\]

\[
P^f = (1 - n)^\frac{1}{\bar{\theta}} \int_{n}^{1} \frac{1}{n} \frac{1}{n} \frac{P^f(z)}{C^f(z)} \frac{1}{\mu} \frac{1}{2} \frac{1}{n} \mu \, dz
\]

\[
P^m = n^\frac{1}{\bar{\theta}} \int_{0}^{1} \frac{1}{n} \frac{1}{n} \frac{P^m(z)}{C^m(z)} \frac{1}{\mu} \frac{1}{2} \frac{1}{n} \mu \, dz
\]

\[
P^m = (1 - n)^\frac{1}{\bar{\theta}} \int_{n}^{1} \frac{1}{n} \frac{1}{n} \frac{P^m(z)}{C^m(z)} \frac{1}{\mu} \frac{1}{2} \frac{1}{n} \mu \, dz
\]
where $P(z)$ is the domestic price of a home produced good $z$; $P^h(z)$ is the foreign price of a home produced good $z$, $P^f(z)$ is the domestic price of a foreign produced good $z$ and $P^{nf}(z)$ is the foreign price of a foreign produced good $z$.

We allow the law of one price to hold so that, if $S$ denotes the nominal exchange rate then, for any product $z$, $P(z) = SP^h(z)$ and $P^f(z) = SP^{nf}(z)$:

This implies, by the definition of the price indexes, that $P^h = SP^h$ and $P^f = SP^{nf}$ and the PPP holds:

$$P = SP^h$$

The individual demands of home and foreign consumers are functions of the relative prices, the elasticities of substitution and the consumption indexes:

$$C^h(z) = \frac{\mu}{P^h} \frac{P(z)}{P^h} \frac{\mu}{P} \frac{P^h}{P} C \ ; \ z \ 2 \ 0; n$$

$$C^f(z) = \frac{\mu}{P^f} \frac{P^f(z)}{P^f} \frac{\mu}{P} \frac{P^f}{P} C \ ; \ z \ 2 \ n; 1$$

$$C^{nh}(z) = \frac{\mu}{P^{nh}} \frac{P^n(z)}{P^{nh}} \frac{\mu}{P^n} \frac{P^{nh}}{P^n} C^n \ ; \ z \ 2 \ 0; n$$

$$C^{nf}(z) = \frac{\mu}{P^{nf}} \frac{P^{nf}(z)}{P^{nf}} \frac{\mu}{P^n} \frac{P^{nf}}{P^n} C^n \ ; \ z \ 2 \ n; 1$$

where we have already included the assumption of within country symmetry of consumers, so that $C(z) = C$ and $C^n(z) = C^n$. Then $C; C^n$ are the aggregate per capita consumptions of each country.

With the above individual demands we can compute the global demand for an individual good $z$:

$$Y(z) = \frac{\mu}{P^h} \frac{P(z)}{P^h} \frac{\mu}{P} \frac{P^h}{P} C^w \ ; \ z \ 2 \ 0; n$$

$$Y^n(z) = \frac{\mu}{P^{nh}} \frac{P^n(z)}{P^{nh}} \frac{\mu}{P^n} \frac{P^{nh}}{P^n} C^w \ ; \ z \ 2 \ n; 1$$

where $C^w$ is the world per capita consumption, defined as:

$$C^w = nC + (1 - n)C^n$$
2.2 Linearized equations

As the model is well-known I will proceed with the linearized equations (see appendix A for the details). All non-capital variables are log-deviations from their steady state values (except for the interest rates which are the deviations from their steady state values).

\[ b_t = (1 + R)b_{t-1} + p_t i + y_t i + \pi_t i + \pi_t^e + \pi_t^f \]  \hspace{1cm} (3)

\[ n b_t = i n b_{t-1} + p_t^e i + p_t^f \]  \hspace{1cm} (4)

\[ q_t = \pi_t^h + s_t i + \pi_t^f \]  \hspace{1cm} (5)

\[ y_t = p_t \]  \hspace{1cm} (6)

\[ (1 + \pi_t^h) b_t = (1 + \pi_t^f) \]  \hspace{1cm} (7)

\[ (1 + \pi_t^h) p_t^e = (1 + \pi_t^f) \]  \hspace{1cm} (8)

\[ c_t = c_{t+1} + \beta c_{t+1} \]  \hspace{1cm} (9)

\[ m_t = m_{t+1} + \pi_{t+1} \]  \hspace{1cm} (10)

The first two equations are the expressions for the current account, including in the second one that \( n b_t + (1 - n) b_t^e = 0 \), where \( b \) is the net foreign asset position. Together they imply the global resource constraint \( c_w = y_w \), where \( c_w = n c_t + (1 - n) c_t^e \) and \( y_w = n(p_t^h + y_t i + \pi_t i) + (1 - n)(p_t^e + y_t^e i + \pi_t^f) \).

Equation (5) defines the terms of trade. Note that in this model the PPP prevails, so that the real exchange rate is constant, but the terms of trade is not. Equation (6) comes from subtracting the global demands faced by producers of both countries. The parameter \( \pi \) turns out to be the critical one in this model, as it controls the magnitude of the expenditure switching effect after a change in the terms of trade. Equations (7) and (8) are the optimal prices for the monopolists. They say that, ceteris paribus, an increase in the CPI will be matched with an equal increase in the monopolist price, that an increase in world consumption (a shift in demand) will increase the price and an increase in home consumption will increase the marginal disutility of...
supplying an additional unit, and thus the monopolist will increase the price. Equation (9) follows from substracting the consumption Euler equations in each country, taking into account that the real interest rate will be the same in each country due to the uncovered interest parity (UIP) and the PPP, as explained in the appendix A and below. Equations (10) and (11) are the money market equilibrium conditions. Finally, (12) and (13) are the loglinearized versions of the CPIs.

The 11 equations (3)-(13) determine the 11 variables: \( y; y^*; c; c^*; p; p^*; p^h; p^{df}; q; s; b \) given the stochastic processes for \( m; m^* \) in (14)-(15). The shocks to the monetary aggregates are permanent.

With this model the neutrality of money holds in the long and short run. Only prices (including the exchange rate) will increase if the money stock increases.

2.3 One period rigid prices

As a rst step towards a dynamic model with price stickiness we construct a one period fix price version. Henceforth we have to distinguish between the optimal prices given in (7)-(8), which we will refer to as \( p^h; p^{df} \):

\[
\begin{align*}
    p^h_t &= p_t + \frac{1}{1 + \lambda_c} c_t + \frac{1}{1 + \lambda_w} c^*_w \\
    p^{df}_t &= p^{*}_t + \frac{1}{1 + \lambda_c} c^*_t + \frac{1}{1 + \lambda_w} c^*_w
\end{align*}
\]

and the actual prices set by firms: \( p^h; p^{df} \). These optimal prices do not hold in the short run, but they do in the long run.

To capture that prices are set one period in advance and adjusted every end of period, we assume the following pricing rules:

\[
\begin{align*}
    p^h_t &= E_{t-1} p^h_t \\
    p^{df}_t &= E_{t-1} p^{df}_t
\end{align*}
\]

The solution of the model is explained in appendix A. The exchange rate is determined combining the conditions for money market equilibrium in both countries:

\[
s_t = (m_t - m^*_t) i_t (c_t - c^*_t) \]

The terms of trade follows from the pricing equations and the above equation for the exchange rate:

\[
q_t = i_t^* + (c_t - c^*_t) i_t \frac{1}{1 + \lambda} - (c^*_t - c^*_t) (c^*_t - c^*_t)
\]
The terms of trade will revert to the initial steady state value after a monetary shock unless there is a permanent difference in per capita consumptions. This difference is in turn:

\[ c_t - c_t^* = \frac{1}{\gamma_{eb}} b_{t+1} + \frac{1}{\gamma_{ec}} (c_t - c_t^*) + \frac{1}{\gamma_{e}} c_t^{*} \]  

(19)

The impact of the monetary shock is determined by the parameter \( \gamma_{e} \) which is the same one calculated in the OR model:

\[ \gamma_{e} = \frac{R(\frac{\gamma}{2} - 1)}{R(1 + \frac{\gamma}{2})} \]

This parameter turns out to be very small: for our baseline parameter values \( \gamma = 1.5; n = 0.5; R = 0.01 \) is only 0.004, meaning that a 1% increase in money supply creates a gap of a 0.004% (of the steady state per capita consumption) between both countries. This gap is related to the wealth effect of the current account surplus that will provide a net foreign asset position permanently higher than the initial:

\[ c_t - c_t^* = \frac{1}{\gamma_{eb}} b_{t+1} \]

where the letter without time subscript stands for long run deviations. However, this effect is very small: in our case \( \frac{1}{\gamma_{eb}} = 0.016 \).

From the pricing rules we can compute the world CPI:

\[ p_{t+1}^w = n p_t + (1 - n) p_{t+1}^* = E_{t+1} p_{t+1}^w + \frac{2}{1 + \frac{\gamma}{2}} E_{t+1} c_{t+1}^{w} \]  

(20)

This expression implies that there is no impact effect on \( p_{t+1}^w \) and that

\[ E_{t+1} c_{t+1}^{w} = 0 \]  

(21)

so that the expected value of global per capita consumption for the period after the shock is the steady state value, because all prices will adjust to the new monetary conditions at that moment. This in turn implies in the Euler equation that

\[ c_t^w = r_t \]  

(22)

\(^3\)Chari, Kehoe and McGrattan (1998) report that the elasticity of substitution between home and foreign goods tends to be between 1 and 2, and they choose a value of 1.5 following Backus, Kehoe and Kydland (1994).
Roughly speaking, a decrease of one point in real interest rate will increase global per capita consumption 1%.

We show in the appendix that aggregating the money market equilibrium conditions and taking into account the pricing rules, we are able to compute $c^w_t$ as:

$$c^w_t = m^w_t \quad \text{and} \quad p^w_t = n^n_t$$

which shows that the nominal shock will increase global consumption but only in the period when the innovation takes place. National per capita consumptions can be obtained as follows:

$$c^e_t = c^w_t + (1 \cdot n)(c_t \cdot c^w_t)$$

$$c^{\nu}_t = c^w_t \cdot n(c_t \cdot c^w_t)$$

From these expressions it can be concluded that if the wealth effect is small, as we have seen is the case, both consumptions will increase practically the same amount, thus producing the high (almost perfect) correlation between home and foreign consumption.

The global resource constraint requires that:

$$c^w_t = y^w_t = n(p^h + y_i \cdot p) + (1 \cdot n)(p^{de} + y^w_i \cdot p^n) = ny + (1 \cdot n)y^n$$

and the national outputs can be computed as:

$$y_t = y^w_t + (1 \cdot n)(y_t \cdot y^n_t) = n^n_t \cdot i \cdot n, q_t$$

$$y^{de}_t = y^w_t \cdot n(y_t \cdot y^n_t) = n^n_t + n, q_t$$

where the approximation comes from $(c_t \cdot c^w_t)' = 0$. While home output will always increase after a monetary expansion, the effect on foreign output will be negative if $\nu > 1$. This implies a negative correlation of $(y; y^n)$ and also a negative correlation of $(y^n; c^n)$. However, both correlations are positive in the business cycle facts and in the VAR evidence.

The results are the same for the short and long run as in OR and in Tille (2000). An increase in $m$ generates an increase of $y$, but also a decrease in $y^n$. Figure 1 has the results of a simulation with the baseline parameter values $n = 0.5$ and $\nu = 1.5$. The increase in $m$ is 1%. The nominal exchange rate jumps immediately to its new steady state value. The home and foreign producer prices are fixed in the first period, but the CPI will increase due to the exchange rate depreciation. The terms of trade worsens due to the
depreciation, but that generates a demand switching effect in favour of home goods, so that home production increases and the foreign one decreases. The consumption increases because of the fall in the real interest rate (and this increase is the same in both countries because they face the same interest rate). As the rise in the income of home \((p^h + y - p)\) is bigger than in consumption, due to consumption smoothing, the current account experiences a superavit. In period 2 the producers’ prices adjust fully to the new conditions, thus reaching the new steady state values. This makes the terms of trade to return to its initial level. The CPIs adjust as well, turning the real interest rate back to the initial value, eliminating the increase in consumption and returning the outputs to normal. The only permanent effect is the one in the net foreign assets, that will increase for the home country, and the interest payments on this positive balance will allow this country to consume forever a little more than initially (the opposite is true for the foreign country).

Tille (2000) shows that the cross-country elasticity controls the sign of the effect of a shock on the welfare of both countries. When this elasticity is small (less than the within country, \(\gamma < \mu\); which is the most probable case) a worsening in the terms of trade will result in a negative welfare effect, thus a monetary expansion would be “beggar thyself” and “prosper thy neighbour”. The OR case is one in which both cross-country and within-country elasticities are the same \(\gamma = \mu\), and this implies that the effects on welfare are positive and of the same magnitude in both countries.

A special case is considered by Corsetti and Pesenti (1999). They show that if the cross-country elasticity is equal to one \(\gamma = 1\), then there are no current account imbalances between the two countries, as the decline in the terms of trade following a monetary shock is offset by a rise in relative nominal income. In this special case there is no wealth reallocation effect and therefore no long run real effect. The worsening of the terms of trade has a negative impact on the welfare of the country that could prove to be bigger than the positive effect of the increase in consumption, bringing about a “beggar thyself” result. The output transmission of the monetary shock could be positive or negative, depending on the elasticity of intertemporal substitution. However, the welfare of the foreign country always increases due to the rise in consumption, thus resulting in a “prosper thy neighbour” effect.

The focus of our attention here is that the correlation of outputs is negative, contrary to the general evidence provided by the international business cycle literature and, more significantly, contrary to the evidence presented in Betts and Devereux (2000) and Kim (1999) from VAR models constructed to analyze the transmission of monetary shocks from one country to another. As discussed in the introduction, they find a positive transmission in outputs.
We wish to make the point that if the monetary authorities in both countries follow an interest rule with a price level target, then they will automatically damp fluctuations in the nominal exchange rate, and that will reduce the expenditure switching effect, allowing the foreign country to increase also its output in response to a home nominal shock (which in that case will be a negative innovation in home nominal interest rate). This is what we will show in the next section.

3 Monetary policy rules

Monetary policy rules have received a great deal of attention since the proposal of Taylor (1993). The recent volume edited by Taylor (1999) contains much of the state of the art on the issue. Interestingly, in that volume only the paper by Ball (1999) considers an open economy, and it is not an optimizing model. Optimizing models of inflation targeting in an open economy are Svensson (2000a) and McCallum and Nelson (1999), both of them deal with the case of a small economy. McCallum and Nelson build their model on the redux model adding staggered prices, but they do not address the main issue we are concerned with here: the sign of the output transmission of monetary shocks.

3.1 The interest rate rule

When the monetary authority fixes the nominal interest rate directly, the monetary aggregate becomes an endogenous variable which can be determined by the demand for money equation. As we are not concerned with the value of the monetary aggregate, we simply substitute in the previous model the money market equilibrium condition by the following policy rule for the nominal interest rate ($I_t$):

$$I_t = d_0 + d_1(\log P_t - \log P_T)$$

where $P_T$ is the target consumer price index. This rule is analyzed in a closed economy by King and Wolman (1999) and Svensson (2000b) discusses the advantages of price-level targeting compared to inflation-targeting. The price level target is going to be the steady state value ($P^* = P_T$). Then, as $\log P_t \cdot \log P^* = p_t$, it implies that $d_0 = R^* = (I_1^{-1})^* = \bar{\epsilon}$; and the rule can be

\footnote{John B. Taylor maintains a very useful web page on monetary policy rules at www.stanford.edu/~johntayl/PolRullLink.htm}
Finally written as:

\[ i_t = 1_t \, R = d_1 p_t + u_t \]  
\[ u_t = \frac{1}{2} u_{t-1} + \sigma t \]  

where \( u \) is the monetary shock, assumed to be an AR(1) process with persistence \( \frac{1}{2} \) and \( \sigma \) is the white noise innovation. With an interest rate rule, monetary shocks are exogenous stochastic shifts in the feedback rule used by the monetary authority, as defined by Rotemberg and Woodford (1998, p.4). McCallum and Nelson (1999) also study the impulse responses after a shock to the interest rate rule, interpreted as unexpected variations in the interest rate. King and Wolman (1999) assume \( \frac{1}{2} = 0.5 \): In a simple model like this one we can think of the shock \( u \) as a short-cut of a demand shock (government spending, investment, private consumption driven by a stock exchange increases, etc).

For the foreign country we assume the same rule:

\[ i^*_t = d_1 p^*_t + u^*_t \]  
\[ u^*_t = \frac{1}{2} u^*_{t-1} + \sigma^*_t \]  

A temporary increase in the domestic nominal interest rate will result in a depreciation for the foreign country that will increase its CPI and will be met with an increase in the foreign nominal interest rate. Hence, the policy rules will enforce a positive correlation of both interest rates. As long as the prices are not responding fully in the short run, the nominal appreciation for the home country will result in an increase of its terms of trade, and that triggers the switching effect of domestic goods for foreign goods, the mechanism that produces the negative correlation of both outputs. But this mechanism will be reduced in a magnitude proportional to the parameter \( d_1 \).

To complete the model we need to link the nominal interest rate to the real variables. The first link is the Fisher equation relating nominal and real interest rates:

\[ 1 + l_t = \frac{E_t P_{t+1}}{P_t} (1 + R_t) \]

or in the linearized form:

\[ i_t = r_t + E_t p_{t+1} i_t p_t \]  

\[ 5^5 \text{Notice that we could have exploited the usual simplification } 1/(1 + R) \approx 1; \text{ but in our case } 1/(1 + R) = \].
The second link is the uncovered interest parity (UIP):

\[ 1 + I_t = \frac{E_t S_{t+1}}{S_t} (1 + I^n_t) \]

in its linearized form:

\[ -i_t = -i^n_t + E_t S_{t+1} i_s \]  \hspace{1cm} (30)

Together with the PPP (\( p = s + p^n \)) both links imply the real interest rate parity: \( r = r^n \). Finally we can use the consumption Euler equation to include the effects of nominal interest rates in the model:

\[ -r_t = -i_t \ E_t p_{t+1} + p_t = E_t c_{t+1} i_s \]

### 3.2 Solution of the model

The policy rules (25) and (27) imply the following equation:

\[ i_t = i^n_t + d_1 s_t + u_t - u^n_t \]

Plugging this into the UIP yields a rational expectations equation for the nominal exchange rate:

\[ - (d_1 s_t + u_t - u^n_t) = E_t S_{t+1} i_s \]

Taking into account the process for \( u_t \) and making \( u^n_t = 0; \forall t \), the solution is:

\[ s_t = \frac{1}{4} (s_t - u_t) \]  \hspace{1cm} (31)

\[ \frac{1}{4} = i \left( 1 + d_1 \right)^{-\frac{1}{2}} < 0 \]

This equation controls the response and the dynamics of the nominal exchange rate, and given that \( p^n \) and \( p^n d \) are fixed in the first period, this equation gives also the immediate response of the terms of trade. It is clear from equation (31) that the response of the exchange rate is smaller the bigger \( d_1 \) is (i.e. the stronger is the commitment of the monetary authority to the target), and that the volatility of the exchange rate increases with the persistence of the shock. In addition, we can show that \( s_{t+1} = \frac{1}{2}s_t \); so that the persistence of the shock is directly translated into the nominal exchange rate.

The terms of trade is:

\[ q_t = i \left( \frac{1}{2} u^n_t + \frac{1}{1+d_1} (c_{ti} + c^n_{ti}) \right) \]  \hspace{1cm} (32)
where the first term is the impact, depending on the innovation, and the second term is the long run effect, depending on the per capita consumption difference generated by the wealth transfer due to a current account imbalance. This will be bigger the more persistent the monetary shock is, as this will increase the response of the terms of trade. Below is shown that the last term is close to zero, so the important movement is determined by the period innovation.

The per capita consumption difference is:
\[ c_t - c_{\delta t} = \frac{1}{\eta_{c_b}} + \frac{1}{\eta_{c_c}}(c_{t-1|1} - c_{\delta t-1}) + \frac{1}{\eta_{e_{t-1}}^c} \]  
(33)

In our baseline parameterization (with \( \frac{1}{\eta_c} = 0.5 \)) the difference in per capita consumption will increase in a 0.002% after a shock of one point in the domestic nominal interest rate.

The pricing rules are the same as in the previous section, therefore we have again the results (20)-(22). In the appendix we show that world per capita consumption is:
\[ c_{w,t} = (1 + \bar{d}_1)n^\eta_t \]  
(34)

So that a decrease in one point of the home nominal interest rate will increase world per capita consumption in 0.6%. The persistence of this innovation does not add inertia to the consumption, but increases the impact. For national consumptions we find the same result as in (23): a high positive correlation since \((c_{t|1} - c_{\delta t|1})\) is very small.

For national outputs we find now:
\[ y_t = y_{w,t} + (1 - n)(y_{t|1} - y_{\delta t}) = (1 + \bar{d}_1)n^\eta_t + (1 - n)\bar{q}_t \]  
(35)

With a decrease in the domestic rate \(\eta < 0\), home output increases, but foreign output increases only if:
\[ 1 + \bar{d}_1 > 0 \]

To fulfill this condition \(d_1\) must be greater than \((r, \bar{d}_1) = \bar{d}_1 = 1.5\). With our baseline value of \(\bar{d}_1 = 1.5\), the value used in the Taylor rule. It is straightforward to see that the correlation between \(y; y^\eta\) will increase with the value of \(d_1\) and decrease with \(\bar{d}_1\).
Figure 2 presents the results of a simulation with $\gamma = 1.5; \theta = 0.5$ and $d_1 = 1.5$. The decrease in the terms of trade is much smaller now. This smaller decrease in the real price of home goods allows a positive output transmission, since the effect of the increase in demand will be stronger than the expenditure switching effect.

4 Adding persistence

The persistence over time of the real effects of a nominal shock is a key question. The empirical evidence (as surveyed by Walsh, 1998) is that a monetary shock brings about a hump shaped impulse response function on output, with a peak in about 6-8 quarters. A strong persistence (or propagation) mechanism is necessary because, with rational expectations, only the unforeseen part of the monetary rule has real effects, so autocorrelation of the monetary process is not a solution, as it is in the case of productivity shocks.

A real persistence mechanism is already present: the reallocation of wealth between countries following a current account (CA) imbalance. Andersen and Beier (2000) analyze a model with this CA propagation mechanism as the only source of persistence, and show that the dynamic response of the terms of trade to a nominal shock does not match the empirical evidence. They stress the importance of staggered price setting as a nominal propagation mechanism that changes the dynamic response of the economy to nominal shocks, making it more similar to the form observed empirically. However, they conclude that persistence obtained with staggered prices is still not enough, and must be reinforced with another persistence mechanism, as capital accumulation. Indeed, Andersen (1999) shows that in a closed economy model both mechanisms, staggering and capital accumulation, reinforce each other and generate a significant amount of persistence in the endogenous variables.

Chari, Kehoe and McGrattan (1998) nd that staggering is crucial to generate a volatile and persistent response of the exchange rate to monetary shocks, but they conclude that to match the observed persistence in real and nominal exchange rates an unrealistic 12 quarters stickiness is needed. Another important problem is that they can only match the observed positive comovements in consumption and output if the monetary processes in both countries are correlated. If they are independent, the transmission mechanism through trade is weak and generates positive but small comovements. Their model has complete markets and pricing to market features, and it is not clear if these features are conditioning the result.
Kollmann (1999) adds staggering following the Calvo (1983) structure in both prices and nominal wages, along with capital accumulation in a complete business cycle model. His result is that nominal shocks have longer real effects with staggering in both prices and wages than in only one of them. He also finds that after a domestic nominal shock the output of the other country does not decrease but increases due to the fact that quantitatively the income effect and the liquidity effect on the other country are more important.

Betts and Devereux (2000) develop a model with staggering a la Calvo (1983) and capital accumulation in which the PTM feature and the completeness of the asset markets can vary, and they relate these features to the international transmission of policy. The effects of monetary policy on the foreign country depends crucially on the degree of PTM up to a point in which the effects are reversed. When there is no PTM the results are those in the OR setup: the depreciation makes the terms of trade fall, triggering an expenditure switching effect from foreign goods to home goods, and the transmission is negative. This result clearly contradicts Kollmann, and more research is needed. In the PTM case a depreciation has the opposite effect on the terms of trade: as the export prices are set in the foreign currency and the import prices in home currency, when the exchange rate depreciates the real price of home goods in terms of foreign goods (the terms of trade) increases. However, with PTM there is no expenditure switching effect as there is no immediate pass-through from the exchange rate to prices and thus in both countries output increases. The problem is that the model predicts a positive correlation between the terms of trade and output and a positive correlation between nominal exchange rate and the terms of trade (Obstfeld and Rogoxx, 2000).

4.1 Staggered prices a la Calvo.

To include nominal persistence we use the Calvo (1983) structure. We define the price probability of changing the price in one period as \( \bar{o} \). Then, denoting by \( \text{pt}_{ht} \) the price set by a home firm in \( t \), this will be (see Appendix):

\[
\text{pt}_{ht} = (\text{pt}_{ht} - (\text{pt}_{ht} - \text{pt}_{ht+1})) + E_t \text{pt}_{ht+1}
\]

and the aggregate home price level:

\[
\text{pt} = \bar{o} \text{pt} + (1 - \bar{o}) \text{pt}_{i=1}
\]

For the foreign country we derive a similar expression:

\[
\text{pt} = \bar{o} \text{pt} + (1 - \bar{o}) \text{pt}_{i=1}
\]
Figure 3 contains the results of a simulation with the same parameters as before, except that $\frac{1}{2}=0$ from now on, and $\theta = 0.2$, so that the expected time between price adjustments is 5 periods. The long run and the impact effects are as explained in section 3. The difference is the adjustment process, that it is not as smooth as we expected. The reason is that we have in fact a smooth adjustment in $p_h^*$, as stated in (36), but this is only a proportion of the CPI variability, dominated by the movement of the exchange rate. The exchange rate has no persistence because the shock to the interest rule is now a one shot without persistence. In equation (12) for the domestic CPI we can see that, in order to have a sluggish CPI adjustment, a smooth adjustment is needed of the home price, the foreign price and the exchange rate. The last requirement is achieved either with a persistent shock ($\frac{1}{2} > 0$) or with inertia in the monetary policy, which we now introduce.

### 4.2 Interest rules with inertia

In this section we turn to the case in which there is inertia in the monetary policy (for reasons analyzed in Woodford 1999). Then we have:

$$i_t = \delta i_{t-1} + (1 - \delta) i^T_t$$

where $i^T_t$ is defined in the previous rule: $i^T_t = d_1 p_t$. Clarida, Gali and Gertler (2000) and McCallum and Nelson (1999) estimate a value of $\delta$ near 0.8.

Figure 4 depicts the effects. Now all the variables, nominal and real, adjust smoothly. Contrary to the case in Figure 3 where $\delta = 0$; now the CPI adjusts slowly, because the exchange rate also does, due in turn to the slow movement of nominal interest rate. Recall that the shock is transitory, not permanent, so that the forward looking behavior of the exchange rate is reflected in the immediate jump, but afterwards it has to return to the initial steady state value and that is controlled by the nominal interest rate differentials through the UIP. The CPI sluggishness makes the real interest rate to move also slowly, then consumption and then output. The impact effects are bigger, compared to Figure 3. Consumption increases more because the real interest rate is under the long run value for longer. The nominal exchange rate jumps further because of the expected longer time of the gap between home and foreign nominal interest rates and the damping effect of the interest rule is reduced to a fifth with $(1 - \delta) = 0.2$ in equation (38). The total increase in global demand will be split between the two countries following equation (6), and as the increase in the nominal exchange rate on impact is bigger, the consequence is that the difference $(y - y^*)$ increases and the correlation $(y; y^*)$ decreases.
Table 1 presents some results on the sensibility to $\pi$. This behaviour of the monetary authority adds inertia to the real variables: the autocorrelation of the output is a significant 0.42 with $\pi = 0.8$:

The last row of the table shows the relative volatility of the terms of trade compared with the volatility of output. This small values could be viewed as a weakness of this model, since one of the stylized business cycle facts is that the value is around 4 (see for instance Chari, Kehoe and McGrattan 1998). However, in a more detailed business cycle model (similar to Kollmann, 1999) we should specify more carefully the proportion of foreign goods that a country consumes, that is, the degree of openness. Here that degree is simply the relative size of the country, which we have assumed 0.5. The result obtained here could be interpreted as making the volatility of the exchange rate the direct reason for the volatility of output, and so to conclude that there is a case for the desirability of a fixed exchange rate or a monetary union. If we wished to apply this model to the US-EU relations, for instance, we should take into account the limited proportion of the international trade between both countries, probably no more than a 10% of the consumption basket. This would highly restrict the effect of the movements of the terms of trade on the output, increasing the relative volatility of the terms of trade and reinforcing our main point, that the transmission of the monetary shocks is more related to the decrease of the real interest rates in both countries than to the trade.

There is a number of ways to reduce the openness. One possibility is to include nontraded goods in the model, as in Hau (2000) and Obstfeld and Rogoφ (2000). This extension is relevant because in addition to reduce the expenditure switching effect, it breaks the PPP, an uncomfortable feature of the model (see Rogoφ, 1996, for a discussion on the PPP). If the PPP does not hold, neither will hold the real interest parity, and this will allow the national consumptions to be less correlated.

A second way is to introduce a home bias in preference as in Warnock (1999), so that for any relative price, Home consumers buy a bigger proportion of home goods than Foreign consumers. This is shown to reduce also the switching effect of the terms of trade.

5 Conclusions

The main conclusion is that according to our model monetary policy rules targeting the consumer price index would tend to produce a positive output transmission of monetary shocks. The reason is that the rule tends to stabilize the nominal exchange rate, reducing the volatility of the terms of
trade and ..nally damping the expenditure switching e¤ect. The direct pos-
itive e¤ect on global demand after a decrease in real interest rates prevails,
providing the interest rule is stronger enough: the parameter controlling the
response to a change in the CPI has to be suf ciently high. Our simulations
suggest that the value of the famous Taylor rule, 1.5, is enough to achieve
the positive correlation of national and foreign outputs.

The second claim is that the inertia of the nominal interest rates turns
out to be a necessary condition to produce inertia in the real variables after
a shock to the interest rule. Without it, even with staggered prices there is
no real persistence in the model after a one time shock to the interest rate
rule, because the CPI will return quickly to its steady state value, as well as
the exchange rate. But if there is inertia in the interest rate the exchange
rate will jump further and then adjust smoothly, and so will do the CPI and
then the real interest rate, consumption and output.

A Solution of the model

This appendix explains brie‡y the model of Obstfeld and Rogo¤ (1995), with
the quali.cation of Tille (2000), which is the workhorse of this paper.

The individual home resident period budget constraint is:

$$P_t B_t + M_t = P_t (1 + R_{t,1}) B_{t,1} + M_{t,1} + P_t Y_t(z) - P_t C_t + P_t T_t$$

(39)

where $B_{t,1}$ is the (end of period) holdings of the unique real bond traded
between both countries, $R_{t,1}$ is the real interest rate, $M_{t,1}$ is the stock of
domestic money, $Y_t(z)$ is the individual production and $T_t$ is the net tax.
The government maintains in every period a balanced budget:

$$T_t = \frac{M_{t,1} - M_{t,1-1}}{P_t}$$

The nominal interest rate $I_t$ is de.ned in the Fisher relationship:

$$1 + I_t = \frac{E_t P_{t+1}}{P_t} (1 + R_t)$$

where $E_t$ is the expectation operator. The uncover interest parity (UIP):

$$1 + I_t = \frac{E_t S_{t+1}}{S_t} (1 + I^n_t)$$

and the PPP imply that the real interest parity also holds:

$$R_t = R^n_t$$

20
The behaviour of the representative agent is obtained by maximizing the utility function \( U \) in (1) subject to the global demand for his product in (2) and the budget constraint in (39). The first order conditions (FOCs) faced by the home representative agent are (similar conditions hold for the foreign agent):

\[
\begin{align*}
C_{t+1} &= \bar{\mu}(1 + R_t)C_t \\
M_t &= \frac{\mu}{AC_t} I_t - \frac{P_t}{AP_t} \mu_i \frac{\tilde{\mu} C_{t+1}}{P_{t+1}} \phi_i \\
[p_t(z)]^{\mu+1} &= P_t^{h+1} P_t^{f+1} C_t C_t^w
\end{align*}
\]

The equilibrium conditions are:

\[
\begin{align*}
nB_t + (1 - n)B_t^n &= 0 \\
C_t^w &= 0
\end{align*}
\]

where \( Y_t^w \) is the world aggregate per capita real income:

\[
Y_t^w = n P_t^{h+1} Y_t + (1 - n) P_t^{f+1} Y_t
\]

We consider a symmetric steady state where \( B = B^n = 0 \). In the steady state:

\[
\begin{align*}
\Gamma &= \bar{\Gamma} = \frac{1}{\bar{\mu}} \\
\bar{C} &= \bar{C}^d = \bar{Y} = \bar{Y}^d = \frac{1}{\bar{\mu}} \\
\bar{P} &= \bar{P}^h = \bar{P}^f = \bar{\mu} \bar{\Gamma}^{-1} \\
\bar{P}^w &= \bar{P}^{h+1} \bar{P}^{f+1} \bar{C}
\end{align*}
\]

Next we explain the loglinearized equations in (3)-(13). Here \( x \) denotes the log-deviation from steady state of \( X \), except for \( r; i \) which are \( r_t = R_t I_t \bar{R}_t \), and \( i_t = I_t I_t \bar{T}_t \). The equilibrium conditions (43) and (44) are equivalent to the equations (3),(4) in the text where \( b_t \) is defined as \( (B_t I_t) \bar{I} \) since \( B^n = 0 \).

The expressions for the terms of trade \( q \) in (5) and for the price indexes in (12) and (13) are straightforward.
Equation (6) is obtained by subtracting the linearized versions of the demand equations (2):

\[ y_t = [p_t, p_h^t] + c_t^w \]
\[ y_t^\alpha = [p_t^\alpha, p_f^t] + c_t^w \]

and using the definition of \( q \).

Equations (7) and (8) are the linearized versions of the optimal prices in (42).

The linearized consumption Euler equations are:

\[ E_t c_{t+1} = r_t \]
\[ E_t c_{t+1}^\alpha = r_t \]

and subtracting them yields equation (9). Equations (10) and (11) are the linearized versions of (41).

### A.1 One period rigid prices

To solve the model we follow the procedure used by Andersen and Beier (2000).

Subtracting the money market equilibrium conditions and exploiting the PPP:

\[ m_t - m_t^\alpha = (c_t - c_t^\alpha) + \frac{1}{1 + R}(E_t s_{t+1} - s_t) \]

which can be solved for the exchange rate:

\[ s_t = \frac{R}{1 + R}(m_t - m_t^\alpha) + \frac{R}{1 + R}(c_t - c_t^\alpha) + \frac{1}{1 + R}E_t s_{t+1} \]

The solution of this expression is equation (17) in the text.

Subtracting the current account equations (3) and (4) yields:

\[ b_t = (1 + R)b_{t+1} + (1 + n)(y_t - y_t^\alpha)(c_t - c_t^\alpha) + (p_h^t - s_t - p_f^t) \]

Using (5) and (6) we can rewrite it as:

\[ b_t = (1 + R)b_{t+1} + (1 + n)(1 - n)(c_t - c_t^\alpha) \]

eliminating \( q \) with (18) yields:

\[ b_t = (1 + R)b_{t+1} + (1 + n)(1 - n)(1 - n)(c_t - c_t^\alpha) \]

22
Now we are ready to solve for \((c_t, c_t')\). Making the guess in (19) we compute:

\[
E_t(c_{t+1} n c_{t+1}) = \frac{1}{(1 + \frac{1}{1 + n})} b_{t+1} (1 + n) c_{t+1} - \frac{1}{1 + \frac{1}{1 + n}} (c_{t+1} n c_{t+1}) \]

Using the Euler equation (9) and equating coefficients:

\[
\frac{1}{\beta_b} = \frac{R(1 + \frac{1}{1 + n})}{R(1 + \frac{1}{1 + n})} \frac{1}{1 + n} > 0
\]

\[
\frac{1}{\beta_c} = \frac{R(1 + \frac{1}{1 + n})}{R(1 + \frac{1}{1 + n})} > 0 \text{ for } \frac{1}{1 + n} > 1
\]

\[
\frac{1}{\beta_e} = \frac{R(1 + \frac{1}{1 + n})}{R(1 + \frac{1}{1 + n})} > 0 \text{ for } \frac{1}{1 + n} > 1
\]

Note that \(\frac{1}{\beta_e}\) computes the impact effect of the nominal shock, and this is exactly equal to the value computed in OR.

To compute the world per capita consumption we first have to compute the world CPI. This is, using the pricing rules in (16):

\[
p^w_t = n p_t + \frac{1}{1 + \frac{1}{1 + n}} c_t + \frac{1}{1 + \frac{1}{1 + n}} c^w_t + (1 + n) E_{t+1} p_{t+1} + \frac{1}{1 + \frac{1}{1 + n}} c^w_{t+1}
\]

Forwarding one period this expression and taking expectations:

\[
E_t p^w_{t+1} = E_t p^w_t + \frac{2}{1 + \frac{1}{1 + n}} E_t c^w_{t+1}
\]

which implies that \(E_t c^w_{t+1} = 0\). Substituting back we end: \(p^w_t = E_t n p^w_t\).

This expectations are formed observing the behaviour of the world money markets, to which we now turn.

Aggregating the money market equilibrium conditions:

\[
m^w_t = p^w_t = c_t^w n p_t\]

Using the previous result that \(E_t c^w_{t+1} = 0\) and rearranging:

\[
p^w_t = \frac{R}{1 + R} m^w_t + \frac{1}{1 + R} E_t p^w_{t+1}
\]
The solution for this equation is:

\[ p_t^w = m_t^w \cdot c_t^w \]

From this, we compute that \( E_t (1 + p_t^w) = m_t^w \cdot 1 \). This implies that:

\[ c_t^w = m_t^w \cdot p_t^w = m_t^w \cdot E_t (1 + p_t^w) = m_t^w \cdot m_{t_i}^w = n_t \]

**B Solution with interest rate rules**

The CA equations imply (45) and using the expression for \( q \) in (32) yields:

\[ b_t = (1 + R) b_{1, i} (1 + n)(c_t^i \cdot c_t^{i^2}) i (1 + n) \frac{1}{1 + \lambda} (c_t^1 i c_t^{i^2}) \frac{i (1 + n)(1 + n)}{1 + \lambda} \frac{1}{1 + \lambda} \]

Now, with the guess in (33) we can compute:

\[ E_t (c_{t+1} i c_{t+1}^n) = \frac{1}{1 + \mu} (1 + R) b_{1, i} (1 + n)(c_t^i \cdot c_t^{i^2}) + (1 + n) \frac{1}{1 + \lambda} (c_t^1 i c_t^{i^2}) \frac{i (1 + n)(1 + n)}{1 + \lambda} \frac{1}{1 + \lambda} \frac{1}{1 + \lambda} \]

Making use of the Euler equation (9) and equating coefficients:

\[ \frac{1}{\lambda_{eb}} = \frac{R (1 + \lambda)}{R (1 + \lambda) + 2, 1 + \lambda} > 0 \]

\[ \frac{1}{\lambda_{ec}} = \frac{R (1 + \lambda)}{R (1 + \lambda) + 2, 1 + \lambda} < 0 \text{ for } \lambda > 1 \]

\[ \frac{1}{\lambda_{ec}} = \frac{R (1 + \lambda)}{R (1 + \lambda) + 2, 1 + \lambda} < 0 \text{ for } \lambda > 1 \]

Aggregating the linearized Fisher equations we have:

\[ -\bar{r}_t = -\bar{i}_t w \cdot E_t \bar{p}_{t+1}^w + \bar{p}_t^w \]

Using (22) and solving for \( \bar{p}_t^w \):

\[ \bar{p}_t^w = \frac{1}{1 + \frac{1}{d_2}} E_t \bar{p}_{t+1}^w \]

The solution of which is:

\[ \bar{p}_t^w = \frac{1}{1 + \frac{1}{d_1}} c_t^w + \frac{1}{1 + \frac{1}{d_1}} u_t^w \]
and so $E_t p_t^w = \frac{1}{2} u_{t-1}^w$ and $E_t p_{t+1}^w = \frac{1}{2} u_t^w$. Substituting $p_t^w = E_t p_t^w = \frac{1}{2} u_{t-1}^w$ and $E_t p_{t+1}^w = \frac{1}{2} u_t^w$ in (46) we finally arrive at the solution for $c_t^w$ in the text:

$$c_t^w = (1 + \bar{d}_1) \frac{1}{2} u_{t-1}^w = (1 + \bar{d}_1) \frac{1}{2} u_t^w$$

C Staggering a la Calvo

Let $\bar{o}$ be the constant probability of adjusting the price at any period. The agent will choose a price $p_{h_t}$ so as to minimize:

$$\frac{1}{2} (p_{h_t} - p_{h_{t-1}})^2 + \frac{1}{2} (1 - \bar{o}) \sum E_t (p_{h_t} - p_{h_{t-1}})^2 + \frac{1}{2} (1 - \bar{o}) \sum E_t (p_{h_t} - p_{h_{t+2}})^2 + \cdots$$

$$\sum_{j=0}^{\infty} \frac{1}{2} (1 - \bar{o})^{j-1} E_t (p_{h_t} - p_{h_{t+j}})^2$$

The FOC for this problem is:

$$p_{h_t} = [1 - (1 - \bar{o})^{-1}] \sum_{j=0}^{\infty} (1 - \bar{o})^{j-1} E_t p_{h_{t+j}}$$

which can be written as:

$$p_{h_t} = [1 - (1 - \bar{o})^{-1}] p_{h_t}^{\text{ha}} + (1 - \bar{o})^{-1} E_t p_{h_{t+1}}^{\text{ha}}$$

With a large number of agents, $\bar{o}$ is also the fraction of them adjusting their prices each period, hence the aggregate home price level evolves according to the equation:

$$p_t^h = \bar{o} p_t^{\text{ha}} + (1 - \bar{o}) p_{t-1}^h$$

References


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$^6$We draw on Walsh (1998) for this procedure.


Table 1. Sensitivity to ®

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Note: Results are the average of 100 simulations of a 100 periods each.
In brackets the standard errors of the coefficients.
Parameter values: ½= 0; = 1.5; ° = 0.2; d1 = 1.5:
Figure 1: Impulse responses after a permanent monetary shock to the domestic stock of money. $\sigma = 1:5$: 
Figure 2: Impulse responses after a persistent shock to the domestic interest rate rule. $d_1 = 1.5$; $\omega = 1.5$; $\gamma = 0.5$.
Figure 3: Impulse responses after a one-time shock to the domestic interest rate rule in the model with staggered prices à la Calvo. \( d_1 = 1.5; \, \, \gamma = 1.5; \, \, \theta = 0.2; \)
Figure 4: Impulse responses after a one-time shock to the domestic interest rate rule in the model with staggered prices à la Calvo and inertia. $d_1 = 1.5$, $\varphi = 1.5; \delta = 0.2; \gamma = 0.8$.