UNIFORM OUTPUT SUBSIDIES IN AN ECONOMIC UNION WITH FIRMS HETEROGENEITY

Bernardo Moreno
José L. Torres

October 2000
DEFI 00/06
Uniform output subsidies in an economic union with firms heterogeneity

Bernardo Moreno\textsuperscript{y} and José L. Torres\textsuperscript{z}
Department of Economics
University of Málaga, Spain
September 28, 2000

Abstract

In this paper we show the importance of cost asymmetry and demand curvature in the effect of a uniform output subsidy policy in an economic union. We consider an economic union formed by two countries each with a single firm producing a homogeneous good. We find that when firms have different cost, the optimal level of the uniform subsidy can be negative if the demand is concave enough. The low cost firm expands its market share if the demand function is sufficiently convex whereas in the case of a concave demand function it is the higher cost firm which gains market share. This implies that a uniform output subsidy policy may cause a change in production efficiency. Finally, we consider how a divergence between private and social costs of public funds may affect the desirability of such a subsidy policy.

Key words: Uniform output subsidy policy, economic union, social welfare, cost differences.

JEL classification: F13, F15

\textsuperscript{x}We are very grateful to Rafael Moner, Robert Hine, Katharine Wakelin, Zhihao Yu, and participants at the VI Jornadas de Economia Internacional, Valencia and GLM Seminars at the University of Nottingham for helpful comments.

\textsuperscript{y}Department of Economics, Facultad Ciencias Económicas y Empresariales, El Ejido s/n, 29013 Málaga, Tlf: +34-952131247, Fax: +34-952131299, e-mail: bernardo@uma.es

\textsuperscript{z}Department of Economics, Facultad Ciencias Económicas y Empresariales, El Ejido s/n, 29013 Málaga, Tlf: +34-952131247, Fax: +34-952131299, e-mail: jtorres@uma.es
1 Introduction

This paper focuses on the implications of uniform output subsidies as a welfare-maximising policy in an imperfectly competitive market. We consider the case of an economic union as an area formed by diferent countries or regions in which there is a common economic authority that decides the same level of subsidies for all the .rms. We assume the existence of two countries each with a single .rm. Firms produce a homogeneous good with constant return to scale and compete in quantities. The objective of the common economic authority is to maximize the social welfare and we assume that the only instrument that it can use is an output subsidy. The question is how to do it. It is easy to see that a carefully chosen subsidy per unit of output to each .rm is efective in achieving an optimal outcome. However, it may be politically infeasible (it will be di¢ cult to justify diferent output subsidies for each country) or technically di¢ cult to apply diferential subsidies.

The use of output subsidies have been widely studied in the context of strategic trade policy by Brander and Spencer (1985), De Meza (1986), Neary (1994) and Collie (1993). Brander and Spencer (1985) were the .rst to show that, in a Cournot duopoly setting, an export subsidy to a home .rm is desirable because it raises the .rm’s market share and pro..ts at the expense of its foreign competitor. Further analysis of .rm asymmetries can be found in De Meza (1986) and Neary (1994). Neary (1994) considers the problem in which the social cost of public funds exceeds unity, nding that they are optimal only for surprisingly low values of the social cost of public funds. Van Long and Soubeyran (1997) show that if domestic .rms do not have identical costs, the optimal trade policy is determined by the interplay between the Her..ndahl index of concentration and the elasticity of demand. For an excellent survey on strategic trade policy see Brander (1995).

In this paper we consider the existence of an economic union with two producing countries with a .rm in each country. Countries (.rms) 1 and 2, produce a homogeneous good for the common market. We assume that there is no trade of the good with the rest of the world, i. e. all production is consumed in the domestic market, in order to study in pure form the efect of the output subsidy. We assume the existence of a common economic

\footnote{For instance, the European Union has attempted to harmonise all aid being given by the governments of member states to the diferent industrial sectors. This was done to eliminate or reduce distortion of competition within the Community.}

\footnote{Note that our model is similar to the third-maket model of Brander and Spencer}
authority that introduces an output subsidy. The economic authority has no other objective than to maximize the social welfare taking both producers and consumers into account. Hence, in this case, the economic authority chooses a certain level of subsidies, the same for the two firms, in terms of welfare maximization. We want to answer the following two questions: i) which is the optimal level of the uniform output subsidy policy? ii) which are the effects of the uniform output subsidy policy on output, market share and payoffs under firm heterogeneity?

We find that the uniform output subsidy level can be negative (a tax) under firm heterogeneity when the demand function is sufficiently concave. Assuming that the uniform output subsidy level is positive, we also find that the optimal uniform output subsidy policy can affect firms differently depending on their cost differences and on the curvature of the demand function. We distinguish three possibilities. First, for sufficiently convex demand functions, the more efficient firm increases its market share and the output differential with respect to the less efficient firm. Inside this region, for some cases the less efficient firm decreases its output. Second, for not too convex demand functions, both firms increase their output, the output differential increases but the less efficient firm increases its market share. Therefore, the output increases of the less efficient firm is proportionally greater than the one of the more efficient firm. Third, for concave demand functions, the output differential decreases, the less efficient firm increases its market share and even, if the demand function is sufficiently concave, the more efficient firm decreases its output. In terms of payoffs, the more efficient firm always increase its payoffs with the introduction of the uniform output subsidy. The more striking conclusion is that when the cost differences are sufficiently large and in the case of convex demand functions, there exists situations for which the less efficient firm decreases its payoffs and therefore the less efficient firm prefers no output subsidies!

Finally, we consider the (more realistic) possibility that the social cost of public funds exceeds unity. We find that the shadow price of public funds depends positively on the cost differences between the firms.

The structure of the paper is as follows. In Section 2 we present the model and the optimal uniform output subsidy in the economic union and its effects in terms of output, market share and profits of the two firms. Section 3 (1985) considering an economic union formed by the two producing countries plus the third (consuming) country.
considers the existence of a cost asymmetry between private and social costs of public funds. Section 4 concludes the paper.

2 The model

We will concentrate on a quantity-setter model. There is a homogenous good produced by firms that have no objectives other than profits. It is well-known that under symmetry, the introduction of a subsidy increases social welfare and firms’ payoffs. However, in general the level of output subsidies for each firm depends on the elasticity, on the degree of concavity of the inverse demand function and on the technology. In order to study when it is social welfare improving to introduce a uniform output subsidy when we have different firms we will assume that we have two firms, 1 and 2, with constant marginal cost functions denoted by \( C_1(x_1) = cx_1 \) and \( C_2(x_2) = \bar{c}x_2 \), where \( c > 0 \) and \( \bar{c} < 1 \), i.e., without loss of generality we assume that firm 1 is more cost competitive than firm 2. We will assume that in each country or region it is located a firm. Let \( p \) be the market price of the product and \( p(x) \) be the inverse demand function mapping aggregate output into prices where \( x \) is total output. We assume that the decision on subsidies is irreversible and prior to decision of firms on output. This situation is modeled as a two-stage game. In stage 1, the social planner chooses a subsidy level per unit of output. In stage 2, firms simultaneously choose output levels for the common market. Let \( s \) be the uniform subsidy per unit of output received by the firms where firm \( i \)'s payoff functions is given by

\[
U_i(x_i; x; s) = p(x)x_i - C_i(x_i) + sx_i \tag{1}
\]

and the consumer surplus is given by

\[
U_3(x) = V(x) - px \tag{2}
\]

where \( V(x) \) is a strictly increasing function.

Definition 1 A Cournot equilibrium is a list of outputs \((x^n_1, \ldots, x^n_N)\) such that for all \( i = 1, 2 \) we have that

\[
p(x^n_i)x^n_i - C_i(x^n_i) + sx^n_i, \ p(x^n_i x^n + x_i)x_i - C_i(x_i) + sx_i \quad 8x_i \neq 2^+\]
From now on we will assume that the Cournot equilibrium is interior. Social welfare (denoted by $W$) is the sum of consumer and producers’s payoff functions net of the value of the subsidy payments

$$W = V(x) + C_1(x_1) + C_2(x_2)$$  \hspace{1cm} (3)

Thus, we can define an optimal allocation as follows:

**Definition 2** A level of subsidy $s^o$ is optimal if it maximizes

$$V(x(s)) + C_1(x_1(s)) + C_2(x_2(s))$$  \hspace{1cm} (4)

Our first assumption restricts the payoff function of each firm to be twice continuously differentiable ($C^2$):

**Assumption 1.** $U_i(x) \in C^2$ for all $i = 1, 2$.

In order to simplify notation, when the context is clear, we will denote derivatives by primes, i.e. $\frac{\partial p(x)}{\partial x} = p_0(x)$, etc. The next assumption requires, on the one hand, that the inverse demand function be either concave or “not too” convex and, on the other hand, it bounds the degree of economies of scale.

**Assumption 2.** $2 + R \leq 0$ where $R = \frac{\partial^2 p(x)}{\partial x^2}$ is a measure of the degree of concavity (convexity) of the inverse demand function.

Finally, the next assumption means that the inverse demand function $p(x)$ is strictly decreasing, i.e. that $V(x)$ is strictly concave.

**Assumption 3.** $U_i(x)$ is strictly decreasing on $x$ given $x_1$ and $x_2$, for all $i = 1, 2$.

Differentiating the order condition of profit maximization for firms with respect to individual output and subsidies we obtain:

$$\frac{dx_i}{ds} = i \frac{p_0(x) (1 + \frac{\partial}{\partial x} R)}{p_0(x) (2 + \frac{\partial}{\partial x} R)} + 1$$  \hspace{1cm} (5)

where $\frac{\partial}{\partial x} = \frac{dx}{dx}$ is the market share of firm $i$. Solving the above system of equations, we get:

$$\frac{dx_i}{ds} = i \frac{1 + (\frac{\partial}{\partial x} R)}{p_0(x) (3 + R)}$$  \hspace{1cm} (6)

and
\[
\frac{dx}{ds} = i \frac{2}{p_0(x) (3 + R)}
\] (7)

From expression (6) it follows that \( \text{rm}_i \)'s change in output depends on the differences in market share between both \( \text{rms} \) in the case of non-linear demand functions. In the case of convex demand functions, \( \text{rm}_i \)'s change in output is larger as the market share of this \( \text{rm} \) is greater. In the case of concave demand functions, \( \text{rm}_i \)'s change in output is lower as the market share of this \( \text{rms} \) is greater. Note also that total output increases with the introduction of the uniform subsidy.

The social welfare, defined as the sum of the consumer and producers' payoffs functions net of the subsidy cost, is given by

\[
W = V(x) i \alpha_1 \alpha_2
\] (8)

Differentiating the social welfare with respect to \( s \), we get:

\[
\frac{dW}{ds} = i \frac{1}{p_0(x) (3 + R)} [(2p(x) i \alpha_1 \alpha_2) R (\beta_1 \beta_2) c(-i, 1)]
\] (9)

Note that the sign of \( \frac{dW}{ds} \) in expression (9) depends on the sign of the expression in brackets. From the first order condition of profit maximization for \( \text{rms} \) we know that the first term of the expression in brackets is positive. Therefore, the change in the social welfare depends on the degree of concavity (convexity) of the inverse demand function and on the differences in cost between the \( \text{rms} \). Social welfare increases with the introduction of subsidies if the demand function is convex or linear (or for concave demand functions if the \( \text{rms} \) are sufficiently similar). This is due to the fact that in the case of convex demand functions the gains in consumer surplus from lower prices are large compared to the cost of the subsidy. In the case of concave demands and \( \text{rms} \) sufficiently different, we have that the gains in consumer surplus are small relative to the subsidy cost.

From expression (6) and the first order condition of the welfare maximization problem we get the following expression for the optimal level of the uniform subsidy:

\[
s = \alpha p(x) \frac{1}{i (\beta_1 \beta_2)^2 R \#} > 0 \quad i \pi \quad R < \frac{1}{(\beta_1 \beta_2)^2}
\] (10)
where \( x = \frac{\partial p(x)}{\partial x} \) is the elasticity of the inverse demand function. As it can be observed, the equilibrium uniform subsidy is positive for convex or linear demand functions. Note also that the uniform subsidy is always positive if firms are symmetric in costs. However, if the demand is concave (\( R > 0 \)), and there are differences in costs such as \((\beta_1 - \beta_2)^2 R > 1\), expression (10) is negative (a tax).

Figure 1 shows the locus for a zero output subsidy as a function of the differences in cost and the curvature of the demand function. If the firms are sufficiently similar, the optimal level of subsidy is positive but for very concave demand function. On the contrary, if the firms are sufficiently different the optimal level of subsidy is negative (a tax) for positive low values of \( R \).

We now study the effects of the uniform output subsidy policy over the firms in terms of outputs, market shares and profits under the assumption that the optimal level of subsidy is positive, i.e. either the demand function is not too concave or the cost differences between the firms are not too large or both. This policy shifts out the best-response functions of both firms, and then, total output increases, prices are driven downwards and therefore the consumer surplus increases. However, if firms have different cost functions we can expect an asymmetric effect of this common policy. In fact, the introduction of the subsidies may increase the output of the less efficient firm and even increase its market share. In this case, the economy as a whole would produce less efficiently. The next proposition presents the results in terms of the output of the firms.

**Proposition 3** In the case of convex demand functions (\( R < 0 \)), the introduction of the uniform subsidy provokes an increase on the output of the lower cost firm and an increase on the differences in output between the firms. In the case of concave demand functions (\( R > 0 \)), the introduction of the subsidy provokes an increase on the output of the higher cost firm and a decrease on the differences in output between the firms. For linear demands (\( R = 0 \)), the introduction of the subsidy increases the output of the two firms by the same amount.

\(^3\)The same result was obtained by Van Long and Soubeyran (1997) when analysing the optimal trade policy in the case of domestic firms with non-identical unit costs.
Proof. From expression (6) we have that the change in output of rm 1 (the low cost rm) is given by:

\[
\frac{dx_1}{ds} = i \frac{1 + (\bar{p}_2 - \bar{p}_1) R}{p^0(x)(3 + R)} \tag{11}
\]

From A.3 and A.2', the denominator in the above expression is negative. Since \((\bar{p}_2 - \bar{p}_1) < 0\), the above expression is positive for convex or linear demand functions. In the case of concave demand functions, expression (11) is also positive for \(R(\bar{p}_2 - \bar{p}_1) < 1\). Similarly, for rm 2 (the high cost rm) we have:

\[
\frac{dx_2}{ds} = i \frac{1 + (\bar{p}_1 - \bar{p}_2) R}{p^0(x)(3 + R)} \tag{12}
\]

where the above expression is positive for concave or linear demand functions. In the case of convex demand functions, expression (12) is positive for \(R(\bar{p}_2 - \bar{p}_1) < 1\). From the first order conditions of profit maximization for rms we have that the difference in output between the rms is given by:

\[
x_1 - x_2 = \frac{c(- \bar{p}_i 1)}{p^0(x)} \tag{13}
\]

Differentiating the output differential with respect to the level of subsidy we get:

\[
\frac{d(x_1 - x_2)}{ds} = \frac{c(- \bar{p}_i 1) R}{p^0(x)} \frac{dx}{ds} \tag{14}
\]

Since \(\frac{dx}{ds} = i \frac{2}{p^0(x)(3 + R)} > 0\) and \(p^0(x) < 0\), the sign of the above expression depends on the curvature of the inverse demand function. The output differential increases (decreases) for convex (concave) demand functions and it does not change for linear demand functions.

In the next proposition, we present the results in terms of the market shares of the rms.

Proposition 4 The introduction of the uniform subsidy increases the market share of the low cost rm if \(R < \bar{p}_1\), increases the market share of the high cost rm if \(R > \bar{p}_1\) and does not change the market share of the rms for \(R = \bar{p}_1\).
Proof. From the first order conditions of profit maximization for firms we have that the difference in market share between the firms is given by:

$$\delta_1 - \delta_2 = \frac{\bar{c}(\gamma - \delta)}{p^0(x)} x$$ \hspace{1cm} (15)

Differentiating the market share differential with respect to the level of subsidy we get:

$$\frac{d(\delta_1 - \delta_2)}{ds} = \frac{\bar{c}(\gamma - \delta)}{p^0(x)} x^2 [R + 1] \frac{dx}{ds}$$ \hspace{1cm} (16)

Since $$\frac{dx}{ds} = i \frac{2}{p(x)(3+R)} > 0$$ and $$p(x) < 0$$, the sign of the above expression depends on the curvature of the inverse demand function. The market share differential increases (decreases) for $$R < \gamma$$ ($$R > \gamma$$) demand functions and it does not change for $$R = \gamma$$.

The above two propositions shows that the optimal uniform output subsidy policy can affect firms differently depending on their cost differences and on the curvature of the demand function. In Figure 2 we present the changes in output and market share of the firms as a function of the curvature of the demand function. We distinguish three regions. For $$R < -1$$, the more efficient firm increases its market share and the output differential with respect to the less efficient firm. Inside this region, if the demand function is sufficiently convex, the less efficient firm decreases its output. For $$-1 < R < 0$$, both firms increase their output, the output differential increases but the less efficient firm increases its market share. Therefore, the output increases of the less efficient firm is proportionally greater than the one of the more efficient firm. For $$R > 0$$, the output differential decreases, the less efficient firm increases its market share and even, if the demand function is sufficiently concave, the more efficient firm decreases its output.

Proposition 5 The introduction of the uniform subsidy increases the payoffs of the low cost firm, whereas the payoffs of the high cost firm may decrease for sufficiently convex demand functions if the firms are sufficiently different.

Proof. The change in the payoffs of firm 1 (the low cost firm) with respect to the level of subsidy is given by:
\[ \frac{dU_1}{ds} = \frac{\partial U_1}{\partial x_1} \frac{dx_1}{ds} + \frac{\partial U_1}{\partial x_2} \frac{dx_2}{ds} + \frac{\partial U_1}{\partial s} \tag{17} \]

From the \( \text{rm} \) 1’s first order condition of profit maximization we have that \( \frac{\partial U_1}{\partial x_1} = 0 \) and provided that \( \frac{\partial U_1}{\partial x_2} = p(x)x_1 \) and \( \frac{\partial U_1}{\partial s} = x_1 \), we get:

\[ \frac{dU_1}{ds} = x_1 \frac{2 + R + R (\frac{\partial U_1}{\partial x_2} \cdot \frac{\partial U_1}{\partial s})}{3 + R} > 0 \tag{18} \]

where the above expression is always positive for any value of \( R \). Therefore, the payoffs of the low cost \( \text{rm} \) always increase with the introduction of the uniform subsidy. In the case of \( \text{rm} \) 2 (the high cost \( \text{rm} \)) we get:

\[ \frac{dU_2}{ds} = x_2 \frac{2 + R + R (\frac{\partial U_1}{\partial x_1} \cdot \frac{\partial U_1}{\partial s})}{3 + R} < 0 \tag{19} \]

where this expression can be negative if the demand function is sufficiently convex and the cost differences between the \( \text{rm} \) s are sufficiently high.\]

From Proposition 5 it can be drawn two conclusions. The more efficient \( \text{rm} \) always increase its payoffs with the introduction of the uniform output subsidy. The more striking conclusion is that when the cost differences are sufficiently large and in the case of convex demand functions, there exists situations for which the less efficient \( \text{rm} \) prefers no output subsidies!

Finally, Propositions 3, 4 and 5 provide a simple explanation of the change in social welfare due to the introduction of the subsidy. The uniform output subsidy policy in the economic union produces two effects. On the one hand, the so-called policy coordination effect, which implies that total output increases, prices are driven downwards and therefore the consumer surplus increases. On the other hand, the so-called homogenization effect, which implies that the introduction of the output subsidy changes the output differential between the \( \text{rm} \) s and their market shares changing the level of production efficiency in the economy as a whole. From Proposition 4 we know that if \( R > \frac{1}{1} \), the introduction of the subsidy makes the less efficient \( \text{rm} \) to increase his market share. Therefore, the economy as a whole produces less efficiently. On the contrary if \( R < \frac{1}{1} \) the more efficient \( \text{rm} \) expands its market share and then, the optimal subsidy policy causes an increase in the total efficiency level of the union. If the cost differences between \( \text{rm} \) s are low, the technological effect disappears. As the cost differences become larger, the technological effects also becomes more important. The sum of these two effects explains why the optimal policy in the economic union is a
tax when the demand is sufficiently concave and cost asymmetry is large. In this situation an output subsidy causes a reduction in the total production efficiency level of the economic union together with the fact that the gains in consumer surplus are small relative to the subsidy cost when the demand is concave. Our results in terms of the optimal subsidy policy are different to the ones obtained by Van Long and Soubeyran (1997) due to the fact that consumers surplus is a component of our definition of the social welfare.

3 Private versus social cost of subsidies

In the preceding analysis private and social costs of public funds have been treated as equivalent. However, in the real world, raising subsidy revenue imposes distortionary costs on the economy, implying that the opportunity cost of a dollar of public funds might exceed 1. Neary (1994) considers the problem of a profit-shifting export subsidy in the third-market model in which the social cost of public funds exceeds unity. He finds that export subsidies are optimal only for surprisingly low values of the social cost of public funds (4/3 for a linear demand, with an upper limit of 2 for non-linear demands) and, if subsidies are justified, they should be higher the more cost competitive the domestic firms are.

Following Neary (1994) we consider a weight parameter $\pm$ to the subsidy payments that may exceed unity, that is, the government places a greater weight on subsidy expenses than on private profit generation. Therefore, the social welfare of the union is defined as:

$$W = V(x) - \alpha x_1 - \bar{\alpha} x_2 - (\pm - 1)sx_1 + (\pm - 1)sx_2 \quad (20)$$

where $\pm \geq 1$; is the relative weight of subsidy cost. As we can observe, as $\pm$ increases, the cost in term of welfare of the output subsidy increases. If $\pm$ is sufficiently high, the optimal policy would be a tax instead of an output subsidy. Several arguments regarding this kind of asymmetry can be considered. First, Gruenspecht (1988) argues that it may reflect the deadweight cost of raising taxes elsewhere in the economy. Brander (1995) notes that

Browning (1987) and Carmichael (1991) give estimations of the parameter $\pm$ for the US economy. Browning (1987) under the assumption that subsidies are financed by tax on labour earnings obtains a value of $\pm$ between 1.10 and 4.03, with preferred estimates lying between 1.32 and 1.47. Carmichael (1991) obtains an estimate of 1.34 for credit subsidies for the export of Boeing 737-200 aircraft.
this will be the case when the government puts less weight on shareholders’ welfare than on taxpayer’s welfare for income distribution or other reasons. Second, the asymmetry between the private and social cost of subsidies can arise from the fact that domestic ...rms can be foreign-owned. Neary (1991) gives a third explanation of ± as the limited budget available to a public agency charged with allocating subsidies between a number of ...ms.

Differentiating the social welfare function with respect to the subsidy yields:

\[
\frac{dW}{ds} = \frac{\partial V}{\partial x} \frac{dx_1}{ds} - \frac{\partial V}{\partial x} \frac{dx_2}{ds} (\pm 1)x_1 (\pm 1)s \frac{dx_3}{ds} = 0
\]  

(21)
or equivalently:

\[
[p_i - c_i (\pm 1)s] \frac{dx_1}{ds} + [p_i - c_i (\pm 1)s] \frac{dx_2}{ds} (\pm 1)x_1 (\pm 1)x_2 = 0
\]  

(22)

Substituting expression (6) in equation (22) and from the rst order condition of pro...t maximization for ...ms we get:

\[
[i \ p^i(x)x_1 \ i (\pm 1)s] \frac{h}{R(\frac{\oplus_1}{\oplus_2})^{\frac{1}{1}}} + [i \ p^i(x)x_2 \ i (\pm 1)s] \frac{h}{R(\frac{\oplus_1}{\oplus_2})^{\frac{1}{1}}} = 0
\]  

(23)

After some simplifications, we obtain the following general expression for the optimal level of subsidy:

\[
s = i \ p^i(x) \frac{4 i \ 3 \pm + R \ i \ \pm R \ i \ R(\frac{\oplus_1}{\oplus_2})^2}{2 \pm}
\]  

(24)

If ± = 1, the above expression reduces to expression (10). Therefore, under symmetry between the social and private costs of subsidies, the optimal subsidy level in an economic union is positive, except if the demand is too concave and the costs differences are su...ciently high. However, we can observe that as ± rises above unity, the optimal output subsidy in the economic
union is more likely to be negative. Following Neary (1994), we consider ...rst the linear case. In the case of a linear demand $R = 0$: The optimal subsidy reduces to:

$$ s = \int p(x) x^4 \left( \frac{3 \pm 2}{2 \pm} \right) $$

which is positive if and only if $\pm < 4 = 3$. Therefore, in the linear case we obtain the same result as Neary (1994) for the non-cooperative export subsidies. In Neary's (1994) terminology, a subsidy is only justified for firms which are more than 75 percent owned by domestic residents.

For non-linear demand, we can show that the threshold value of $\pm$ at which the optimal subsidy switches from a positive to a negative value is:

$$ \pm^* = \frac{4 + R \left( \frac{\bar{\gamma}_1}{\bar{\gamma}_2} \right)^2}{3 + R} $$

Comparing $\pm^*$ with the threshold value in the linear case ($4 = 3$), we obtain:

$$ \pm^* \frac{4}{3} = \frac{4 R \left( \frac{\bar{\gamma}_1}{\bar{\gamma}_2} \right)^2}{3 + R} $$

The denominator is always positive since $2 + R > 0$. The numerator will be negative for concave demand and positive for convex demand. From this set of results we obtain the following two propositions:

**Proposition 6** The threshold value of $\pm$ is lower than $4 = 3$ for concave demand ($R > 0$) and greater than $4 = 3$ for convex demand ($R < 0$).

**Proof:** In the case of a concave demand, the numerator of expression (27) is always negative. On the other hand, in the case of a convex demand, the numerator is always positive.

**Proposition 7** In the case of symmetric production costs ($\gamma = 1$), the maximum value for $\pm$ is 2. As asymmetries in production costs increase, the value increases, with a maximum value for $\pm$ of 4.

**Proof:** In the symmetric case, $\bar{\gamma}_1 = \bar{\gamma}_2 = 1=2$, so the expression reduces to

$$ \frac{i R}{3 + R} $$

13
Taking the limit when $R$ goes to $\sqrt{2}$, we obtain an upper limit value of $\pm 2$. As $\mathcal{R}_1 \cdot \mathcal{R}_2$ goes to one, the expression reduces to:

$$\frac{4R}{9 + 3R}$$

(29)

For a value of $R = \sqrt{2}$, the upper limit of $\pm 4$ is 4, that is, the output subsidy is positive when the government values a dollar of its revenue between one and four dollars of private payoffs. This value is twice that obtained by Neary (1994) for the non-cooperative profit-shifting export subsidy case.

Figure 3 shows the threshold value of the social cost of public funds as a function of the differences in costs for some particular values of the curvature of the demand function. If the demand is concave, the threshold value decreases as differences in costs increases. If the demand is convex, the threshold value increases as differences in costs increases. Therefore, in our case the output subsidy is likely to be more positive than in the standard non-cooperative game when the demand is convex and firms have different costs. This high value for the social cost of public funds is explained by the fact that when differences in costs are very high and the demand is convex, the subsidies cause large gains in consumer surplus relative to the subsidy cost. Ballard et al. (1985) suggest an opportunity cost in the range of 1.17 to 1.56 per dollar raised for the US economy. The maximum value that we obtain is larger than the empirically plausible values.

[Insert here Figure 3]

4 Conclusions

In this paper we show the importance of cost asymmetry and demand elasticity in the effect of a uniform output subsidy policy for the case of an economic union in which there are two firms producing a homogeneous good. We assume that the union is self-sufficient in the good and that there is a common economic authority whose objective is to maximize social welfare. In so doing, it takes account of the interests of both firms and consumers in the union, using the sole instrument at its disposal: an output subsidy. We find that the optimal output subsidy level may be negative (i.e. a tax) if cost differences are important and the demand is concave.

This policy causes two effects: the homogenization effect and the policy coordination effect. The first effect causes an increase in total production
in the union, a fall in price and then, an increase in consumer surplus. The second effect implies that the uniform output subsidy policy in the economic union causes a change in the production efficiency level of the economy of the union as a whole. We find that when $R < \frac{1}{2}$, the low cost firm expands its market share whereas for $R > \frac{1}{2}$ it is the higher cost firm which expands its market share. Therefore, this policy affects the level of production efficiency in the economic union. However, we show that, in general, the two firms increase their profits, except when the demand is convex and cost differences are important. In the latter case, the uniform subsidy policy causes a reduction in the profits of the less efficient firm. This set of results shows the importance of firms heterogeneity across countries and the curvature of the demand functions in the design of an output subsidy policy in an economic union (i.e. this can be the case of the European Union).

Finally, we consider an asymmetry between private and social costs, that is, we take into account that the social cost of public funds can be larger than unity. We find that in the case of a linear demand the threshold shadow price of public funds for the output subsidy to be positive is 4/3. This value increase as asymmetries in costs are larger and the demand is more convex, with a maximum value of 4. This value is larger than the empirical estimated values, suggesting that even after taking account of the distortionary effect of raising taxes, an output subsidy policy could be welfare improving in an imperfectly competitive market in the economic union context.

References


Figure 1. Locus for a zero uniform output subsidy as a function of the curvature of the inverse demand function and market share differences.
Figure 2. Effects on output and market shares of the uniform output subsidy policy.
Figure 3. Threshold values of the social cost of the uniform output subsidy as a function of market share differences.