Price-Cost Margins and Economic Integration: How Important is the Pro-Competitive Effect?

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PRICE-COST MARGINS AND ECONOMIC INTEGRATION: HOW IMPORTANT IS THE PRO-COMPETITIVE EFFECT?*

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Abstract
In this paper we examine whether the conventional result of a greater degree of integration leading to lower price-cost margins (i.e., the pro-competitive effect), would hold when two countries integrate by forming a common market. We propose a general framework of reference, in order to assess the extent of the pro-competitive effect when the role of other variables is allowed for, both for a “small” and “large” common market. By solving the model, the price-cost margin of domestic firms would depend on a set of variables in addition to trade costs with the partner country, which might eventually offset the conventional result.

JEL classification: F15, F12

Key words: Economic integration, pro-competitive effect, price-cost margins

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1. Introduction
The so-called “pro-competitive” effect, i.e., the disciplinary effect of an increased foreign competition on domestic markups, stands as one of the main potential outcomes of a process of economic integration, according to models of international trade with imperfect competition (Baldwin and Venables, 1995). In particular, the higher levels of efficiency and welfare, due to this reduction in market power, have been mentioned as one of the most important benefits to be reached following the implementation of the Single Market Programme in the European Union (EU); see, among others, Flam (1992), Allen, Gasiorek and Smith (1998), or Bottasso and Sembenelli (2001).

The rationale behind this conclusion would be as follows. The removal of trade barriers associated with a process of economic integration would mean that the size of the relevant market for domestic firms is now greater, so that their market shares would decrease. As a consequence, domestic firms would reduce their price-cost margins, and then their ability of charging higher prices at home than abroad. In addition, the increased competition would entail some industrial restructuring, through the entry of new firms into the enlarged market and the exit of the less efficient producers, as well as a greater exploitation of economies of scale. All this would result in fewer distortions, lower prices, and a higher level of welfare (Allen, Gasiorek and Smith, 1998).

Some authors, however, have challenged this “conventional wisdom”. For instance, Haaland and Wooton (1992) notice that a process of integration does not necessarily amount to enlarging the market and hence reducing market power in the previously domestic markets. In particular, they conclude that the probability for prices to rise, instead of falling, after integration would be greater the higher were trade costs, the bias in preferences towards domestic goods, and the degree of concentration in the market (i.e., the lower the number of firms).

The purpose of this paper is to examine whether the conventional result of a greater degree of integration leading to lower price-cost margins, and hence falling prices, would hold, when two countries integrate by forming a common market. More specifically, we propose a general framework of reference, in order to assess the extent of the pro-competitive effect when the role of other variables is allowed for. To that end, we develop
a simple model of pricing behaviour in an imperfectly competitive industry, with three
types of firms: home, partner or associated, and foreign. The common market formed by
the home and partner countries is assumed to be either “small” or “large”, namely, when
foreign variables are taken to be exogenous or endogenous, respectively. By solving the
model, the price-cost margin of domestic firms would depend on a set of variables in
addition to trade costs with the partner country. In this way, we would be able to
establish whether the expected decrease in the price-cost margin of domestic firms
might be offset by eventual changes in these other variables.

The paper is structured as follows. In section 2, we present a simple model of price
setting in a competitive industry, where the relationship between the price-cost margin of
domestic firms and its potential explanatory factors should be highlighted. Some
concluding remarks are presented in section 3.

2. The model

We will develop in this section a simple, partial equilibrium model of an industry where
firms compete à la Cournot. The model incorporates three types of firms: home (i.e.,
those from the domestic country), partner or associated (i.e., those from the country
forming a common market with the domestic country) and foreign (i.e., those from the
rest of the world), denoted by subscripts $h$, $a$ and $f$, respectively. Each firm produces a
variety of a differentiated good, and, for simplicity, all firms are assumed to be of equal
size; there are $n_h$, $n_a$ and $n_f$ home, partner and foreign firms, respectively.

Product differentiation is modelled according to the approach of Dixit and Stiglitz
(1977). The representative consumer in each country maximizes her utility, given by the
quantity index:

$$Q = \left[ \frac{1}{\sigma} n_h q^\sigma_h + \frac{1}{\sigma} n_a q^\sigma_a + \frac{1}{\sigma} n_f q^\sigma_f \right]^{\alpha \sigma^{-1}}$$

where $q$ is the quantity consumed of each type of variety, $\sigma>1$ is the elasticity of
substitution among varieties, and $\alpha$ is a parameter indicating the extent of idiosyncratic
tastes [extending Warnock’s (1998) specification to the case of three types of varieties].
The $\alpha$’s are normalized to add one, so that no bias in preferences would appear when $\alpha_h$
= \alpha = \alpha_f = \frac{1}{3}; we will assume a bias in preferences towards home over partner goods, and towards partner over foreign goods, which would occur when \alpha_h > \frac{1}{3}, and \alpha_h > \alpha_a > \alpha_f. The price index dual to (1) is:

\[ P = \left( \alpha_h n_h p_h^{1-\sigma} + \alpha_a n_a p_a^{1-\sigma} + \alpha_f n_f p_f^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \]  

(2)

where \( p \) is the price of each type of variety.

Given the budget constraint:

\[ Y = PQ = n_h p_h q_h + n_a p_a q_a + n_f p_f q_f \]  

(3)

where \( Y \) is the (fixed) total expenditure on differentiated products, maximization of (1) subject to (3) leads to the following demand function for every type of variety:

\[ q_i = \alpha_i p_i^{-\sigma} P^\sigma Q \quad i = h, a, f \]  

(4)

whose inverse function is:

\[ p_i = \alpha_i^{\frac{1}{\sigma}} q_i^{-\frac{1}{\sigma}} Q^{\frac{1}{1-\sigma}} Y \quad i = h, a, f \]  

(5)

Now, we can find from (5) the (absolute value of the) perceived elasticity of demand, \( \eta_i \), for every type of variety, by first computing its inverse:

\[ \frac{1}{\eta_i} = -\frac{\partial p_i}{\partial q_i} = \frac{1}{\sigma} + \left( \frac{\sigma - 1}{\sigma} \right) s_i \quad i = h, a, f \]

so that:

\[ \eta_i = \frac{\sigma}{1 + (\sigma - 1)s_i} \quad i = h, a, f \]  

(6)

where \( s_i = \frac{p_i q_i}{Y} \) is the market share for each type of firm \( (i = h, a, f) \), and

\[ n_h s_h + n_a s_a + n_f s_f = 1. \]

Profit-maximizing firms set prices as a markup over marginal costs. Denoting by \( c_i \) \( (i = h, a, f) \) the firm’s marginal cost, this can be shown in the case of home firms as:

\[ p_h = \left( \frac{\eta_h}{\eta_h - 1} \right) c_h \]

which, after replacing \( \eta_h \) from (6), becomes:
\[ p_h = \left( \frac{\sigma}{(\sigma - 1)(1 - s_h)} \right) c_h \]  \hspace{1cm} (7)

Following a similar reasoning, the pricing behaviour of partner firms can be expressed as:
\[ p_a = \left( \frac{\sigma}{(\sigma - 1)(1 - s_a)} \right) c_a t_a \]  \hspace{1cm} (8)

where \( t_a = (1 + \tau_a) \) is the trade cost faced by partner firms, being \( \tau_a \) the *ad valorem* cost.

Finally, regarding foreign firms, and in analogy with the case of a monetary union in open economy macroeconomics (Bajo-Rubio and Díaz-Roldán, 2001), two alternative assumptions can be made on the size of the common market formed by the home and partner countries. More specifically, the common market can be either “small”, so that \( p_f \) would be exogenous; or “large”, in which case:
\[ p_f = \left( \frac{\sigma}{(\sigma - 1)(1 - s_f)} \right) c_f t_f \]  \hspace{1cm} (9)

where \( t_f = (1 + \tau_f) \) is the trade cost faced by foreign firms, being \( \tau_f \) the *ad valorem* cost.

Notice, on the other hand, that market shares can be expressed from (4) and (5) as [see equation (2.4) in Baldwin and Venables (1995, p. 1607)]:
\[ s_i = \alpha_i \left( \frac{p_i}{P} \right)^{1-\sigma} \quad i = h, a, f \]

and, replacing \( P \) from (2) for \( i = h, a, \) and \( f \):
\[ s_h = \frac{\alpha_h p_h^{1-\sigma}}{\alpha_h n_h p_h^{1-\sigma} + \alpha_a n_a p_a^{1-\sigma} + \alpha_f n_f p_f^{1-\sigma}} \]  \hspace{1cm} (10)

\[ s_a = \frac{\alpha_a p_a^{1-\sigma}}{\alpha_h n_h p_h^{1-\sigma} + \alpha_a n_a p_a^{1-\sigma} + \alpha_f n_f p_f^{1-\sigma}} \]  \hspace{1cm} (11)

\[ s_f = \frac{\alpha_f p_f^{1-\sigma}}{\alpha_h n_h p_h^{1-\sigma} + \alpha_a n_a p_a^{1-\sigma} + \alpha_f n_f p_f^{1-\sigma}} \]  \hspace{1cm} (12)

From here, the models for the small and large common market would be as follows.
Small common market

The model for the small common market would be made of four equations: (7), (8), (10), and (11); with four endogenous variables: \( p_h, p_a, s_h, s_a \). Solving the system for \( p_h \), defining the price-cost margin of domestic firms, \( PCM_h \), as:

\[
PCM_h = \frac{p_h}{c_h}
\]  

(13)

and replacing the solution for \( p_h \) in (13), we would get an equation for \( PCM_h \) as a function \( \mu \) of the elasticity of substitution, the bias in preferences towards domestic and partner varieties (remember that \( \alpha_f = 1 - \alpha_a - \alpha_h \)), the number of firms, trade costs for partner firms, the foreign firms’ price, and marginal costs:

\[
PCM_h = \mu(\sigma, \alpha_h, \alpha_a, n_h, n_a, n_f, t_a, p_f, c_h, c_a)
\]  

(14)

In this equation, a regional integration agreement, such as the formation of a common market, would be reflected in a decrease in (and the eventual elimination of) \( t_a \).

Since the \( p \)’s enter in an exponential way in equations (10) and (11), the solution to the system has been found by fully differentiating the model, so that the multipliers associated to each explanatory variable would be elasticities of the price-cost margin with respect to it. This solution is given by:

\[
\frac{dMPC_h}{MPC_h} = \frac{1}{D} \left( N_1 \frac{d\sigma}{\sigma} + N_2 \frac{d\alpha_h}{\alpha_h} + N_3 \frac{d\alpha_a}{\alpha_a} + N_4 \frac{dn_h}{n_h} + N_5 \frac{dn_a}{n_a} + N_6 \frac{dn_f}{n_f} + \\
+ N_7 \frac{dt_a}{t_a} + N_8 \frac{dp_f}{p_f} + N_9 \frac{dc_h}{c_h} + N_{10} \frac{dc_a}{c_a} \right)
\]  

(15)

where: \( D > 0, N_1 > 0, N_2 > 0, N_3 > 0, N_4 < 0, N_5 < 0, N_6 < 0, N_7 = N_{10} > 0, N_8 > 0, N_9 > 0; \) the exact definition of \( D \) and the \( N \)’s is given in the Appendix.

Therefore, according to (15), the price-cost margin of domestic firms would depend on:

- the elasticity of substitution among varieties (\( \sigma \)), with an ambiguous sign;
- the parameters indicating the bias in preferences towards domestic and partner varieties (\( \alpha_h \), and \( \alpha_a \)), with an ambiguous sign;
• the number of (home, partner, and foreign) firms operating in the domestic market \((n_h, n_a, \text{ and } n_f)\), negatively;
• the trade costs faced by partner firms \((t_a)\), positively;
• the price charged by foreign firms \((p_f)\), positively; and
• the marginal costs of (home and partner) producers \((c_h, \text{ and } c_a)\), positively.

**Large common market**

If a large common market is assumed instead, (7) to (12) would form a system of six equations with six endogenous variables, namely, the prices and market shares of the three types of firms. As before, \(PCM_h\) would be a function of the exogenous variables:

\[
PCM_h = \Pi(\sigma, \alpha_h, \alpha_a, n_h, n_a, n_f, t_a, t_f, c_h, c_a, c_f)
\]

where a regional integration agreement would be reflected again in a decrease in (and the eventual elimination of) \(t_a\). Notice that, unlike the small common market case, two new exogenous variables would appear, namely, \(t_f\) and \(c_f\), which would replace \(p_f\).

Solving the system for \(PCM_h\) in a similar way than before, we would get:

\[
\frac{dMPC_h}{MPC_h} = \frac{1}{D} \left( \frac{d\sigma}{\sigma} + \frac{d\alpha_h}{\alpha_h} + \frac{d\alpha_a}{\alpha_a} + \frac{dn_h}{n_h} + \frac{dn_a}{n_a} + \frac{dn_f}{n_f} + \right.

+ \left. \frac{dt_a}{t_a} + \frac{dt_f}{t_f} + \frac{dc_h}{c_h} + \frac{dc_a}{c_a} + \frac{dc_f}{c_f} \right)
\]

where \(D > 0\), \(\bar{N}_1 > 0\), \(\bar{N}_2 > 0\), \(\bar{N}_3 > 0\), \(\bar{N}_4 < 0\), \(\bar{N}_5 < 0\), \(\bar{N}_6 < 0\), \(\bar{N}_7 > 0\), \(\bar{N}_8 = 0\), \(\bar{N}_9 > 0\), \(\bar{N}_{10} > 0\), and the exact definition of \(D\) and the \(\bar{N}\)’s is given again in the Appendix. Hence, the price-cost margin of domestic firms would depend in the large common market case, on:

• the elasticity of substitution among varieties \((\sigma)\), with an ambiguous sign;
• the parameters indicating the bias in preferences towards domestic and partner varieties \((\alpha_h, \text{ and } \alpha_a)\), with an ambiguous sign;
• the number of (home, partner, and foreign) firms operating in the domestic market \((n_h, n_a, \text{ and } n_f)\), negatively;
• the trade costs faced by partner and foreign firms \((t_a, \text{ and } t_f)\), positively; and
• the marginal costs of (home, partner, and foreign) producers ($c_h$, $c_{as}$, and $c_f$), positively.

As can be seen, the results for the small and large common market (leaving aside the more complex coefficients found for the latter case) are fairly analogous, with $t_f$ and $c_f$ replacing $p_f$. Hence, we will centre our comments in the perhaps more realistic case of a large common market (think, e.g., of the Single Market in the EU).

A regional integration agreement would lead, _ceteris paribus_, to an unambiguous fall in the price-cost margin of home firms (i.e., the pro-competitive effect). However, it is possible that this result could be offset by changes in other variables. Leaving aside changes in both the elasticity of substitution and preferences regarding domestic and partner goods, which show an ambiguous effect on the price-cost margin, and should rather occur in the long term, the possibility of a reversion of the pro-competitive effect would be higher if:

• a greater integration with the partner country is accompanied by an increase in the trade barriers borne by firms from the rest of the world;
• the number of (home, partner, and foreign) firms operating in the domestic market decreases; or
• the marginal costs of (home, partner, and foreign) producers increases.

Regarding marginal costs, it does not seem too clear their relationship with integration; and even a reduction in marginal costs, rather than an increase, might be expected following an increase in productive efficiency after integration. More relevant would seem the role of trade barriers against the rest of the world: if a process of integration is to be accompanied with a higher degree of protection towards third countries’ firms, by decreasing the market share of these firms and increasing that of home firms, this would lead to an increase in the price-cost margin of home firms. And the same would happen if the increased competition associated with a process of integration leads to a net exit of firms into the enlarged market. This point would be of a crucial importance, since a process of integration can generate a number of sometimes conflicting effects that might lead to the number of both home and foreign firms to either increase or decrease; see, e.g., the discussion in Markusen and Venables (1999).
Therefore, the effect of integration on the price-cost margin of home firms (i.e., the extent of the pro-competitive effect) would be ambiguous on theoretical grounds, once the possibility of changes in other variables following a process of integration is recognised, and would turn to be an empirical question.

3. Concluding remarks

We have examined in this paper to what extent the conventional result claiming that a greater degree of integration would lead to lower price-cost margins, and hence falling prices, actually holds, when two countries integrate by forming a common market. We have proposed a general framework of reference, in order to assess the extent of the pro-competitive effect when the role of other variables is allowed for. To that end, we have developed a simple model of pricing behaviour in an imperfectly competitive industry, with three types of firms: home, partner or associated, and foreign. The common market formed by the home and partner countries has been assumed either “small” or “large”, namely, when foreign variables were taken to be exogenous or endogenous, respectively. Once solved the model, the price-cost margin of domestic firms depended, in addition to trade costs with the partner country, on several other variables, such as the elasticity of substitution among varieties, the bias in preferences towards domestic and partner goods, the number of firms operating in the domestic market, trade costs with the foreign country, and the marginal costs of firms. In particular, price-cost margins might increase following a process of integration if the latter were accompanied with a higher degree of protection towards third countries’ firms; or, alternatively, if the increased competition would lead to a net exit of firms into the enlarged market.
Appendix

Small common market

\[ D = 1 + (\sigma - 1) \left[ \frac{s_h}{1 - s_h} (1 - n_h s_h) + \frac{s_a}{1 - s_a} (1 - n_a s_a) \right] + (\sigma - 1)^2 \left[ \frac{s_h}{1 - s_h} \frac{s_a}{1 - s_a} (1 - n_h s_h - n_a s_a) \right] \]

\[ N_1 = - \left( \log p_h - \log p_f \right) \frac{s_h}{1 - s_h} \left[ (1 - n_h s_h) + (\sigma - 1) \frac{s_a}{1 - s_a} (1 - n_h s_h - n_a s_a) \right] + \]

\[ + \left( \log p_a - \log p_f \right) \frac{s_h}{1 - s_h} n_a s_a - \frac{1}{\sigma(\sigma - 1)} \left( 1 + (\sigma - 1) \left[ \frac{s_h}{1 - s_h} n_a s_a + \frac{s_a}{1 - s_a} (1 - n_a s_a) \right] \right) \]

\[ N_2 = \frac{s_h}{1 - s_h} \left[ \frac{1 - \alpha_a}{1 - \alpha_h - \alpha_a} (1 - n_h s_h) - \frac{\alpha_h}{1 - \alpha_h - \alpha_a} n_a s_a \right] + \]

\[ + (\sigma - 1) \left[ \frac{1 - \alpha_a}{1 - \alpha_h - \alpha_a} \frac{s_a}{1 - s_a} (1 - n_h s_h - n_a s_a) \right] \]

\[ N_3 = \frac{s_h}{1 - s_h} \left[ \frac{\alpha_a}{1 - \alpha_h - \alpha_a} (1 - n_h s_h) - \frac{1 - \alpha_h}{1 - \alpha_h - \alpha_a} n_a s_a \right] + \]

\[ + (\sigma - 1) \left[ \frac{\alpha_a}{1 - \alpha_h - \alpha_a} \frac{s_a}{1 - s_a} (1 - n_h s_h - n_a s_a) \right] \]

\[ N_4 = - \frac{s_h}{1 - s_h} n_h s_h \left[ 1 + (\sigma - 1) \frac{s_a}{1 - s_a} \right] \]

\[ N_5 = - \frac{s_h}{1 - s_h} n_a s_a \left[ 1 + (\sigma - 1) \frac{s_a}{1 - s_a} \right] \]

\[ N_6 = - \frac{s_h}{1 - s_h} (1 - n_h s_h - n_a s_a) \left[ 1 + (\sigma - 1) \frac{s_a}{1 - s_a} \right] \]

\[ N_7 = N_{10} = (\sigma - 1) \frac{s_h}{1 - s_h} n_a s_a \]
\[ N_8 = (\sigma - 1) \frac{s_h}{1 - s_h} \left( 1 - n_h s_h - n_a s_a \right) + (\sigma - 1)^2 \frac{s_h}{1 - s_h} \frac{s_a}{1 - s_a} \left( 1 - n_h s_h - n_a s_a \right) \]

\[ N_9 = (\sigma - 1) \frac{s_a}{1 - s_a} \left( 1 - n_a s_a \right) \]

Large common market

\[ \bar{D} = 1 + (\sigma - 1) \left[ \frac{s_h}{1 - s_h} \left( 1 - n_h s_h \right) + \frac{s_a}{1 - s_a} \left( 1 - n_a s_a \right) + \frac{s_f}{1 - s_f} \left( n_h s_h + n_a s_a \right) \right] + \]

\[ + (\sigma - 1)^2 \left[ \frac{s_h}{1 - s_h} \frac{s_a}{1 - s_a} \left( 1 - n_h s_h - n_a s_a \right) + \frac{s_h}{1 - s_h} \frac{s_f}{1 - s_f} n_a s_a + \frac{s_a}{1 - s_a} \frac{s_f}{1 - s_f} n_h s_h \right] \]

\[ \bar{N}_1 = - \left( \log p_h - \log p_f \right) \frac{s_h}{1 - s_h} \left( 1 - n_h s_h \right) + (\sigma - 1) \left[ \frac{s_a}{1 - s_a} \left( 1 - n_h s_h - n_a s_a \right) + \frac{s_f}{1 - s_f} n_a s_a \right] + \]

\[ + \left( \log p_a - \log p_f \right) \frac{s_h}{1 - s_h} n_a s_a \left[ 1 + (\sigma - 1) \frac{s_f}{1 - s_f} \right] - \frac{1}{\sigma(\sigma - 1)} \bar{D} \]

\[ \bar{N}_2 = \frac{s_h}{1 - s_h} \left[ \frac{1 - \alpha_a}{1 - \alpha_h - \alpha_a} \left( 1 - n_h s_h \right) - \frac{\alpha_h}{1 - \alpha_h - \alpha_a} n_a s_a \right] + \]

\[ + (\sigma - 1) \left[ \frac{1 - \alpha_a}{1 - \alpha_h - \alpha_a} \frac{s_a}{1 - s_a} \left( 1 - n_h s_h - n_a s_a \right) + \frac{s_f}{1 - s_f} n_a s_a \right] \]

\[ \bar{N}_3 = \frac{s_h}{1 - s_h} \left[ \frac{\alpha_a}{1 - \alpha_h - \alpha_a} \left( 1 - n_h s_h \right) - \frac{1 - \alpha_h}{1 - \alpha_h - \alpha_a} n_a s_a \right] + \]

\[ + (\sigma - 1) \left[ \frac{\alpha_a}{1 - \alpha_h - \alpha_a} \frac{s_a}{1 - s_a} \left( 1 - n_h s_h - n_a s_a \right) - \frac{s_f}{1 - s_f} n_a s_a \right] \]

\[ \bar{N}_4 = - \frac{s_h}{1 - s_h} n_h s_h \left[ 1 + (\sigma - 1) \left( \frac{s_a}{1 - s_a} + \frac{s_f}{1 - s_f} \right) + (\sigma - 1)^2 \frac{s_a}{1 - s_a} \frac{s_f}{1 - s_f} \right] \]

\[ \bar{N}_5 = - \frac{s_h}{1 - s_h} n_a s_a \left[ 1 + (\sigma - 1) \left( \frac{s_a}{1 - s_a} + \frac{s_f}{1 - s_f} \right) + (\sigma - 1)^2 \frac{s_a}{1 - s_a} \frac{s_f}{1 - s_f} \right] \]
\[
\overline{N}_6 = -\frac{s_h}{1-s_h} (1-n_h s_h - n_a s_a) \left[ 1 + (\sigma - 1) \left( \frac{s_a}{1-s_a} + \frac{s_f}{1-s_f} \right) + (\sigma - 1)^2 \frac{s_a}{1-s_a} \frac{s_f}{1-s_f} \right]
\]

\[
\overline{N}_7 = \overline{N}_{10} = (\sigma - 1) \frac{s_h}{1-s_h} n_a s_a + (\sigma - 1)^2 \frac{s_h}{1-s_h} \frac{s_f}{1-s_f} n_a s_a
\]

\[
\overline{N}_8 = \overline{N}_{11} = (\sigma - 1) \frac{s_h}{1-s_h} (1-n_h s_h - n_a s_a) + (\sigma - 1)^2 \frac{s_h}{1-s_h} \frac{s_a}{1-s_a} (1-n_h s_h - n_a s_a)
\]

\[
\overline{N}_9 = (\sigma - 1) \left[ \frac{s_a}{1-s_a} (1-n_a s_a) + \frac{s_f}{1-s_f} (n_h s_h + n_a s_a) \right] + (\sigma - 1)^2 \frac{s_a}{1-s_a} \frac{s_f}{1-s_f} n_h s_h
\]
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