

# **DOCUMENTOS DE ECONOMÍA Y FINANZAS INTERNACIONALES**

## **Working Papers on International Economics and Finance**

**DEFI 10-08  
November 2010**

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Asociación Española de Economía y Finanzas Internacionales  
[www.aeefi.com](http://www.aeefi.com)  
ISSN: 1696-6376

# The Spanish term structure of interest rates revisited: cointegration with multiple structural breaks, 1974-2010\*

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November 2010

## Abstract

In this paper we consider the possibility that a linear cointegrated regression model with multiples structural changes would provide a better empirical description of the term structure model of interest rates. Our methodology is based on instability tests recently proposed in Kejriwal and Perron (2010) as well as the cointegration test in Arai and Kurozumi (2007) and Kejriwal (2008) developed to allow for multiple breaks under the null hypothesis of cointegration.

*Keywords:* Term structure of interest rates; Cointegration; Multiple Structural Breaks

*JEL classification:* C32, E43

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\*Vicente Esteve acknowledges financial support from the regional government of Castilla-La Mancha, through the project PEII09-0072-7392, and the Generalitat Valenciana (Project GVPROMETEO2009-098). The authors acknowledge financial support from the MICINN (Ministerio de Ciencia e Innovación), through the projects ECO2008-05908-C02-02 (Vicente Esteve), SEJ2008-05565/ECON (Manuel Navarro-Ibáñez) and ECO2009-13616 (María A. Prats).

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# 1 Introduction

The expectations hypothesis (EH) of the term structure of interest rates is one of the oldest and simplest analytical framework that simplifies the rational behaviour in the financial markets. The EH of the term structure of interest rates, which states that the observed term structure can be used to infer market participants expectations about future interest rates, has been at the origin of an extraordinary amount of econometric analysis; see, e. g., Campbell (1995), Campbell and Shiller (1987, 1991), Engsted and Tanggaard (1994a,b), Hall et al. (1992), Hardouvelis (1994), Jondeau and Ricart (1999), Lanne (2000), Sarno et al. (2007), Thornton (2006), and Tzavalis (2003).

Understanding the term structure of interest rates has always been viewed as crucial to assess the impact of monetary policy and its transmission mechanism, to predict interest rates, exchange rates and economic activity, and to provide information about expectations of participants in financial markets. However, the term structure of interest rates is likely to be subject to variation as a result of changes in the structure of the economy, like changes in monetary policy or in the exchange rate regime and reforms in the financial market regulation. Thus, the information content of the term structure is subject to change over time and all the empirical modeling work that does not take into account the possible variations and instability will fail to explain the variations in the term structure of interest rates.

According to the EH, the long-term interest rates should reflect future short-term changes. Specifically, long-term interest rates would be the average of future expected short rates. Hence, the EH in the context of the cointegration theory suggests that the long and short interest rates are linked through a long-run relationship with parameters (1, -1), i.e. that the interest rate spread is mean-reverting. Following the work by Campbell and Shiller (1987), a number of further contributions have arisen. These works have strived to test the EH of the term structure of interest rates applying cointegration techniques on a linear model, leading sometimes to contradictory results. A non-exhaustive list of them would include, Stock and Watson (1988), Hall et al. (1992), Engsted and Tanggaard (1994a, b), Cuthbertson (1996).

In an empirical study, Camarero and Tamarit (2002) extended the previous analysis on the expectations model of the term structure of interest rates addressing the question of whether the relationship is stable over time, or exhibit a structural break allowing the instability to occur at an unknown point. For the Spanish economy they found evidence of linear cointegration between long and short interest rates for the period 1980:1-1996:4, with a vector (1, -1) as predicted by the theory. Moreover, the tests for instability and structural change detected the presence of a break in 1994 when two factors that may have affected the term structure of interest rates were acting: first, the successive devaluations of the peseta, that happened at the end of 1982 and between 1992 and 1995; second, the financial changes that occurred at the beginning of 1994, as a result of the commitments of Spain in the context of the process towards the European Monetary Union.

Camarero and Tamarit (2002) applied several methods to detect the structural changes or instability in the cointegration regressions. The first group of tests are those of the null hypothesis of no change in cointegrated models proposed by Hansen (1992). These LM test procedures are based on the fully modified estimation method (Phillips and Hansen, 1990) which has been shown to lead to tests with very poor finite sample properties (Carrion-i-Silvestre and Sansó-i-Roselló, 2006). The results in Quintos and Phillips (1993) also suggest that the LM tests are likely to suffer from the problem of low power in finite samples. Moreover, simulation experiments in Hansen (2000) show that the LM test is quite poorly behaved in the presence of structural changes in the marginal distributions of the regressors. The second group of tests, proposed by Gregory and Hansen (1992a, b), consider the residual-based test for the null of no cointegration against the alternative of cointegration with a structural break of unknown timing. A rejection by these tests would then confirm the presence of a cointegrating relationship with a structural break. However, the value of the break associated with the minimal value of a given statistic is not, in general, a consistent estimate of the break date if a change is present. Moreover, these tests are designed to have power against the alternative of a single break in parameters and hence may have low power when the alternative involves more than one break. The third group of tests are the multiple structural changes tests proposed by Bai and Perron (1998a, b) in the context of OLS recursive estimation applied to stationary variables. However, these tests are only valid for stationary variables and the interest rates series are both  $I(1)$  or non-stationary variables.

In this paper we extend the existing empirical analysis of the term structure model of interest rates in two ways. First, in order to avoid the econometric problems mentioned above, we make use of recent developments in cointegrated regression models with multiple structural changes. Specifically, we use a new approach proposed by Kejriwal and Perron (2008, 2010) to test for multiple structural changes in cointegrated regression models. They develop a sequential procedure that not only enables detection of parameter instability in cointegration regression models but also allows consistent of the number of breaks present. Furthermore, we test the cointegrating relationship when multiple regime shifts are identified endogenously. In particular, the nature of the long run relationship between long and short interest rates is analyzed using the residual based test of the null hypothesis of cointegration with multiple breaks proposed in Arai and Kurozumi (2007) and Kejriwal (2008). Second, a common criticism to most test of the term structure of interest rates is that the econometric procedures used require a large number of observations. Accordingly, in this paper we use a long span of the data (1974:1-2010:2). It will allow us to obtain more robust results on the fulfilling of the term structure of interest rates than in previous analysis.<sup>1</sup>

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<sup>1</sup>A recent empirical study of Esteve (2006) extends the previous analysis on the EH of the term structure of interest rates addressing the possibility that a nonlinear model might provide a better empirical description. This paper applies the methodology to test for threshold cointegration recently proposed by Hansen and Seo (2002) to the Spanish term structure of

The rest of the paper is organized as follows. A brief description of the underlying theoretical framework is provided in section 2, the methodology and empirical results are presented in sections 3 and 4, respectively, and the main conclusions are summarized in section 5.

## 2 A simple model of the EH of the term structure of interest rates

In order to test the term structure of interest rates in the context of the cointegration theory, the empirical studies on the EH have commonly used a linear model such as:

$$bonds_t = c + \gamma cmr_t + \varepsilon_t \quad (1)$$

where  $bonds_t$  is the interest rate of long-term bonds and  $cmr_t$  the short-term interest rate. According to Campbell and Shiller (1987),  $bonds_t$  and  $cmr_t$  should be non-stationary and linked through a cointegration relationship with parameters (1, -1). Campbell and Shiller (1991) noted that this hypothesis implies that a maturity-specific multiple of the term spread predicts future changes in the long bond yield. Thus, the expectations theory of the term structure suggests that the current interest rate spread is an optimal forecast of future changes in long-run interest rates. Thus, according to this hypothesis, market's expectations about the short-rate developments of the bond yield are reflected in the slope of the term structure with a one-to-one relation.

Alternatively, we may write the linear regression model (1) as a bivariate linear cointegrating VAR model with one lag,  $l = 1$ , such as:

$$\begin{pmatrix} \Delta bonds_t \\ \Delta cmr_t \end{pmatrix} = \mu + \alpha w_{t-1} + \Gamma \begin{pmatrix} \Delta bonds_{t-1} \\ \Delta cmr_{t-1} \end{pmatrix} + \varepsilon_t \quad (2)$$

where the long-run relationship is defined as  $w_{t-1} = bonds_{t-1} - \gamma cmr_{t-1}$ . Setting  $\gamma = 1$ , the long-run relationship would be the same as the interest rate spread,  $s_t$ .

## 3 Methodology

### 3.1 A linear cointegrated regression model with multiples structural changes

Issues related to structural change have received a considerable amount of attention in the statistics and econometrics literature. Bai and Perron (1998) and

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interest rates during the period 1980:1-2002:12. The evidence suggests that nonlinear cointegration between long and short interest rates is clearly rejected, so that a linear cointegration model would provide an adequate empirical description for the Spanish term structure of interest rate.

Perron (2006, 2008) provide a comprehensive treatment of the problem of testing for multiple structural changes in linear regression models. Accounting for parameter shifts is crucial in cointegration analysis since it normally involves long spans of data which are more likely to be affected by structural breaks. In particular, Kejriwal and Perron (2008, 2010) provide a comprehensive treatment of the problem of testing for multiple structural changes in cointegrated systems.

More specifically, Kejriwal and Perron (2008, 2010) consider a linear model with  $m$  multiple structural changes (i.e.,  $m + 1$  regimes) such as:

$$y_t = c_j + z'_{ft}\delta_f + z'_{bt}\delta_{bj} + x'_{ft}\beta_f + x'_{bt}\beta_{bj} + u_t \quad (t = T_{j-1} + 1, \dots, T_j) \quad (3)$$

for  $j = 1, \dots, m + 1$ , where  $T_0 = 0$ ,  $T_{m+1} = T$  and  $T$  is the sample size. In this model,  $y_t$  is a scalar dependent I(1) variable,  $x_{ft}(p_f \times 1)$  and  $x_{bt}(p_b \times 1)$  are vectors of I(0) variables while  $z_{ft}(q_f \times 1)$  and  $z_{bt}(q_b \times 1)$  are vectors of I(1) variables.<sup>2</sup> The break points  $(T_1, \dots, T_m)$  are treated as unknown.

The general model (3) is a partial structural change model in which the coefficients of only a subset of the regressors are subject to change. In our case, we suppose that  $p_f = p_b = q_f = 0$ , and estimated model is a pure structural change model with all coefficients of the I(1) regressors and constant (slope and the intercept in (1)) allowed to change across regimes:

$$y_t = c_j + z'_{bt}\delta_{bj} + u_t \quad (t = T_{j-1} + 1, \dots, T_j) \quad (4)$$

Generally, the assumption of strict exogeneity is too restrictive and the test statistics for testing multiple breaks are not robust to the problem of endogenous regressors. To deal with the possibility of endogenous I(1) regressors, Kejriwal and Perron (2008, 2010) propose to use the so-called dynamic OLS regression (DOLS) where leads and lags of the first-differences of the I(1) variables are added as regressors, as suggested by Saikkonen (1991) and Stock and Watson (1993):

$$y_t = c_i + z'_{bt}\delta_{bj} + \sum_{j=-l_T}^{l_T} \Delta z'_{bt-j}\Pi_{bj} + u_t^*, \quad \text{if } T_{i-1} < t \leq T_i \quad (5)$$

for  $i = 1, \dots, k + 1$ , where  $k$  is the number of breaks,  $T_0 = 0$  and  $T_{k+1} = T$ .

### 3.2 Structural Break Tests

In this paper we test the parameter instability in cointegration regression using the tests proposed in Kejriwal and Perron (2008, 2010). They present issues related to structural changes in cointegrated models which allows both I(1) and I(0) regressors as well as multiple breaks. They also propose a sequential procedure which permits consistent estimation of the number of breaks, as in Bai and Perron (1998).

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<sup>2</sup>The subscript  $b$  stands for "break" and the subscript  $f$  stands for "fixed" (across regimes).

Kejriwal and Perron (2010) consider three types of test statistics for testing multiple breaks. First, they propose a sup *Wald* test of the null hypothesis of no structural break ( $m = 0$ ) versus the alternative hypothesis that there are a fixed (arbitrary) number of breaks ( $m = k$ ):

$$\sup F_T^*(k) = \sup_{\lambda \in \Lambda_\varepsilon} \frac{SSR_0 - SSR_k}{\hat{\sigma}^2} \quad (6)$$

where  $SSR_0$  denote the sum of squared residuals under the null hypothesis of no breaks,  $SSR_k$  denote the sum of squared residuals under the alternative hypothesis of  $k$  breaks,  $\lambda = \{\lambda_1, \dots, \lambda_m\}$  is the vector of breaks fractions defined by  $\lambda_i = T_i/T$  for  $i = 1, \dots, m, T_i$ , and  $T_i$  are the break dates.

Second, they consider a test of the null hypothesis of no structural break ( $m = 0$ ) versus the alternative hypothesis that there is an unknown number of breaks, given some upper bound  $M(1 \leq m \leq M)$ :

$$UD \max F_T^*(M) = \max_{1 \leq k \leq m} F_T^*(k) \quad (7)$$

In addition to the tests above, Kejriwal and Perron (2010) consider a sequential test of the null hypothesis of  $k$  breaks versus the alternative hypothesis of  $k + 1$  breaks:

$$SEQ_T(k + 1|k) = \max_{1 \leq j \leq k+1} \sup_{\tau \in \Lambda_{j,\varepsilon}} T \left\{ SSR_T(\hat{T}_1, \dots, \hat{T}_k) \right\} \quad (8)$$

$$- \left\{ SSR_T(\hat{T}_1, \dots, \hat{T}_{j-1}, \tau, \hat{T}_j, \dots, \hat{T}_k) \right\} / SSR_{k+1} \quad (9)$$

where  $\Lambda_{j,\varepsilon} = \left\{ \tau : \hat{T}_{j-1} + (\hat{T}_j - \hat{T}_{j-1})\varepsilon \leq \tau \leq \hat{T}_j - (\hat{T}_j - \hat{T}_{j-1})\varepsilon \right\}$ . The model with  $k$  breaks is obtained by a global minimization of the sum of squared residuals, as in Bai and Perron (1998).

### 3.3 Cointegration tests with structural changes

Kejriwal and Perron (2008, 2010) show that the structural change tests can suffer from important lack of power against spurious regression (i.e, no cointegration). This means that these tests can reject the null of stability when the regression is really a spurious one. In this sense, tests for breaks in the long run relationship are used in conjunction with tests for the presence or absence of cointegration allowing for structural changes in the coefficients.

In this paper, we use the residual based test of the null of cointegration with an unknown single break proposed in Arai and Kurozumi (2007), in which they developed a *LM* test based on partial sums of residuals where the break point is obtained by minimizing the sum of squared residuals. They considered three models: i) Model 1, level shift; ii) Model 2, level shift with trend; iii) and Model 3, regime shift.

The *LM* test statistic (for one break),  $\tilde{V}_1(\hat{\lambda})$ , is given by:

$$\tilde{V}_1(\hat{\lambda}) = (T^{-2} \sum_{t=1}^T S_t(\hat{\lambda})^2) / \hat{\Omega}_{11} \quad (10)$$

where  $\hat{\Omega}_{11}$  is a consistent estimate of the long run variance of  $u_t^*$  in (5), the date of break  $\hat{\lambda} = (\hat{T}_1/T, \dots, \hat{T}_k/T)$  and  $(\hat{T}_1, \dots, \hat{T}_k)$  are obtained using the dynamic algorithm proposed in Bai and Perron (2003).

The Arai and Kurozumi (2007) test is restrictive in the sense that only a single structural break is considered under the null hypothesis. Hence, the test may tend to reject the null of cointegration when the true data generating process exhibits cointegration with multiple breaks. To avoid this problem, Kejiriwal (2008) has recently extended the Arai and Kurozumi (2007) test by incorporating multiple breaks under the null hypothesis of cointegration. The Kejiriwal (2008) test of the null of cointegration with multiple structural changes is denoted with  $k$  breaks as  $\tilde{V}_k(\hat{\lambda})$ .

## 4 An application to the Spanish term structure of interest rates

The data used in this paper are monthly for Spain and cover the period 1974:1 to 2010:2. The variables utilized in the empirical application are the nominal long-term interest rate,  $bonds_t$  (private bonds of electric utilities before February 1978; from March 1978 to December 1992, central government bonds at more than two years; and, from January 1993, central government benchmark bond of 10 years), and the nominal short-term interest rate,  $cmr_t$  (1-month interbank market rates before December 1976; and, from January 1977, 3-month interbank market rates). Both series have been obtained from Bank of Spain (2010). The evolution of the two series is shown in Figure 1 and there seems to be a close comovement between the two series. However, the plots also suggest that the long-short interest rates association may have altered over time.

As a preliminary step in our analysis, we examine the time series properties of the series by testing for a unit root over the full sample. We have used a modified version of the Dickey-Fuller and Phillips-Perron tests proposed by Ng and Perron (2001), which try to solve the main problems present in these conventional tests for unit roots.

In general, the majority of the conventional unit root tests suffer from three problems. First, many tests have low power when the root of the autoregressive polynomial is close to, but less than, unit (Dejong *et al.*, 1992). Second, the majority of the tests suffer from severe size distortions when the moving-average polynomial of the first-differenced series has a large negative autoregressive root (Schwert, 1989; Perron and Ng, 1996). Third, the implementation of unit root tests often needs the selection of an autoregressive truncation lag,  $k$ ; however, as discussed in Ng and Perron (1995) there is a strong association between  $k$  and the severity of size distortions and/or the extend of power loss.



Recently, Ng and Perron (2001) have proposed a methodology that solves these three problems. This method consists of a class of modified tests, called  $\bar{M}_{MAIC}^{GLS}$ , originally developed in Stock (1999) as  $M$  tests, with GLS detrending of the data as proposed in Elliot *et al.* (1996), and using the Modified Akaike Information Criteria (*MAIC*).<sup>3</sup> Also, Ng and Perron (2001) have proposed a similar procedure to correct for the problems of the standard Augmented Dickey-Fuller (ADF) test,  $ADF_{MAIC}^{GLS}$ .<sup>4</sup>

Table 1 reports the results of Ng and Perron tests. This table shows that the nominal long-term interest rate is found to be I(1), while the null hypothesis of nonstationarity for the nominal short-term interest rate can be rejected at the 1% significance level with the  $\bar{M}Z_{\alpha}^{GLS}$ ,  $\bar{M}Z_t^{GLS}$  and  $ADF^{GLS}$  tests. Therefore, according to the results of these tests, the nominal short-term interest rate series could be I(1) or I(0).

A potential difficulty in assessing the time series properties of monetary and financial variables, is that they can be subject to potential structural breaks in the form of infrequent changes in the mean or the drift of the series, due to exogenous shocks or changes in the policy regime. Hence, in order to provide further evidence on the degree of integration of variables, we have also applied the Perron-Rodriguez test (Perron and Rodriguez, 2003) for a unit root in the presence of a one time change in the trend function.

Perron and Rodriguez (2003) extend the tests for a unit root analyzed by Perron and Ng (2001) to the case where a change in the trend function is allowed to occur at an unknown time,  $T_B$ . In this paper we use the method where the break date is selected minimizing the tests, as suggested by Zivot and Andrews (1992). The results are presented in Table 2. We consider the Model II where a structural change in intercept and slope is allowed to occur at an unknown time. Using the *MAIC* to select  $k$ , there is no evidence against the unit root for the nominal short-term interest rate series at the 5% significance level. The break date is selected at 1980:2.

An alternative method to select the break date, as used in Perron (1997), is to choose it such that the absolute value of the  $t$ -statistic on the coefficient of the change in slope is maximized. Table 3 presents the results of the tests. For nominal short-term interest rate series, there is not evidence against the unit root. The break date selected is 1979:9. Therefore, according to the results of these tests,  $cmr_t$  would be I(1).

Once the order of integration of the series has been analyzed, we estimate the long-run or cointegration relationship between  $bonds_t$ , and  $cmr_t$ . Given the relatively small sample size, we will estimate and test the coefficients of the cointegration equation by means of the Dynamic Ordinary Least Squares (DOLS) method from Saikkonen (1991) and Stock and Watson (1993) and following the methodology proposed by Shin (1994). This estimation method provides a robust correction to the possible presence of endogeneity in the explanatory

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<sup>3</sup>These tests are the  $\bar{M}Z_{\alpha}^{GLS}$ ,  $\bar{M}SB^{GLS}$  and  $\bar{M}Z_t^{GLS}$ .

<sup>4</sup>See Ng and Perron (2001) and Perron and Ng (1996) for a detailed description of these tests.

variables, as well as serial correlation in the error terms of the OLS estimation. Also, in order to overcome the problem of the low power of the classical cointegration tests in the presence of persistent roots in the residuals of the cointegration regression, Shin (1994) suggests a test where the null hypothesis is that of cointegration. In the first place, we estimate a long-run dynamic equation including the leads and lags of all the explanatory variables, the so-called DOLS regression; in our case:

$$bonds_t = c + \gamma cmr_t + \sum_{j=-q}^q \gamma_j \Delta cmr_{t-j} + v_t \quad (11)$$

Secondly, the Shin's test is based on the calculation of two LM statistics from the DOLS residuals,  $C_\mu$ , to test for deterministic cointegration. The parameter  $\gamma$  is the long-run cointegrating coefficient estimated between the long and short interest rates (or long-run elasticity).

The results of Table 4 show that the null of deterministic cointegration between  $bonds_t$  and  $cmr_t$  is not rejected at the 1% level of significance, and the estimated value for  $\gamma$  is 0.77, significantly different from zero at the 1% level. But this estimate would be significantly different from one at the 1% level, according to a Wald test on the null hypothesis  $\hat{\gamma} = 1$  against the alternative  $\hat{\gamma} < 1$ , distributed as a  $\chi_1^2$  and denoted by  $W_{DOLS}$  in Table 4. Since the estimate of long-run elasticity is significantly lower than one, so that changes in the long-term interest rate would have not been fully adjusted to compensate the behaviour of the short-term interest rates.

Accounting for parameter shifts is crucial in cointegration analysis, which normally involves long spans of data, which are more likely to be affected by structural breaks. Our data covers thirty five years of the history of the interest rates, during which time the term structure of interest rates have probably changed due to variations in macroeconomic and market forces, such as changes in the structure of the economy, changes in the monetary policy or exchange rate regime, supply shocks, and reforms in the financial and tax regulation. Therefore, as we argued before, it is important to account for structural breaks in our cointegration relationship.

We now consider the tests for structural change that have been proposed in Kejriwal and Perron (2008, 2010). We use 15% trimming so that the maximum numbers of breaks allowed under the alternative hypothesis is 5. Both the intercept and the slope of equation (11) are allowed to change. Table 5 presents the results of stability tests as well as the number of breaks selected by the sequential procedure (SP) proposed by Bai and Perron (2003). The  $UD$  max test is significant at the 5% level, which implies that at least one break is present. The  $\sup F_T(1)$  test is significant at the 5% level, unlike  $\sup F_T(2)$ , suggesting that the data do not support a two-break model. The sequential procedure selects a single break and provide evidence against the stability of the long run relationship. Overall, the results of the Kejriwal-Perron tests suggest a model with one break estimated at 1979:6 and two regimes, 1974:1-1979:6 and 1979:7-

2010:2.<sup>5</sup> The break date 1979:6 is precisely estimated with since their 95% confidence interval cover only a few months before and after (1979:4-1980:11). There are some factors that may explain the placement of such structural change of the Spanish term structure of interest rates.

First, the domestic financial sector had experienced a serious liberalisation process. Until the early 1980s, most financial transactions were going through the banking system, which itself was strongly regulated. In addition to reserve and investment requirements, most interest rates were administered. The numerous regulatory changes produced the development of several financial markets, including the interbank market (linked to the short interest rates), the market for public debt and the stock market (both linked to the long interest rates). Such liberalisation was in line with the various regulations and new financial directives of the EU.

Second, until the early 1980s the deficits of the public sector were financed mostly via credits from the Bank of Spain (seigniorage). Only after 1982, budget deficits were increasingly financed in a more orthodox way using market mechanisms, through the issuing of public debt, which allowed the development of the secondary market for public debt and the use of central government bonds as reference of the long-term interest rates.

Since the above stability tests also reject the null coefficient stability when the regression is a spurious one, we still need to confirm the presence of cointegration among the variables. For this reason we use the residual based test of the null of cointegration against the alternative of cointegration with an unknown single break proposed in Arai and Kurozumi (2007),  $V_1(\hat{\lambda})$ . Arai and Kurozumi (2007) show, in the single break case, that the limit distribution of the test statistic,  $\tilde{V}_1(\hat{\lambda})$ , depend only on the timing of the estimated break fraction  $\hat{\lambda}$  and the number of I(1) regressors  $m$ . In our case, critical values are obtained for  $\hat{\lambda} = 0.15$  and  $m = 1$  by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.*  $N(0, 1)$  random variables. Since we are interested in the stability of the short-term-long-term interest rate coefficient,  $\gamma$ , we consider only model 3 that permits the slope shift as well a level shift. Table 6 shows the results of the Arai-Kurozumi cointegration test with a single break. Again, the level of trimming used is 15%. The results show that the test  $\tilde{V}_1(\hat{\lambda})$  cannot reject the null of cointegration with a structural break at 1979:6.

In order to compare the coefficients obtained from a break model with those reported from a model without any structural break, we proceed to estimate the cointegration equation (11) for the two sub-samples, and the results are shown in the last two columns of Table 4. First, the estimates show that the slope estimated is insignificant in the first regime and the estimated parameter is very small (0.09). Second, the coefficient estimated for the second regime is significant and higher (0.83) than the full sample estimate of 0.77. This suggests that

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<sup>5</sup>Note that this result is very similar to the change selected for the nominal short-term rate series when we apply the Perron-Rodriguez test for a unit root in the presence of a one time change in the trend function (Table 3).

ignoring shifts may understate the long-run cointegration relationship between the long and short interest rates.

## 5 Conclusions

Accounting for parameter shifts is crucial in cointegration analysis, which normally involves long spans of data, which are more likely to be affected by structural breaks. In this paper we consider the possibility that a linear cointegrated regression model with multiples structural changes would provide a better empirical description of the term structure model of interest rates. Our methodology is based on instability tests recently proposed in Kejriwal and Perron (2008, 2010) as well as the cointegration test in Arai and Kurozumi (2007) and Kejriwal (2008) developed to allow for multiple breaks under the null hypothesis of cointegration. This method is applied to test the Spanish term structure of interest rates during the period 1974:1-2010:2.

The results are consistent with the existence of linear cointegration between the long and the short run Spanish interest rates, with a vector  $(1, -0.77)$ . Thus, the cointegration vector is not  $(1, -1)$ , as predicted by the theory. However, our empirical results show also that the cointegrating relationship has changed over time. In particular, the Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models would suggest a model of two regimes, with the dates of the break estimated at 1979:6. The break date 1979:6 is precisely estimated with since their 95% confidence interval cover a only a few months before and after (1979:4-1980:11). In addition, the Arai-Kurozumi-Kejriwal cointegration test with a single structural break cannot reject the null of cointegration with a structural break at 1979:6.

There are factors that may explain the placement of such structural change of the Spanish term structure of interest rates. First, the domestic financial sector had experienced a serious liberalisation process. Until the early 1980s, most financial transactions were going through the banking system, which itself was strongly regulated. In addition to reserve and investment requirements, most interest rates (both short-term and long-term interest rates) were administered. The numerous regulatory changes allowed the development of several financial markets, including the interbank market (linked to short-term interest rates), the market for public debt and the stock market (both linked to long-term interest rates). Such liberalisation implemented from 1980 was in line with the various regulations and new financial directives of the EU.

Second, until early 1980s the public deficits were financed mostly via credits from the Bank of Spain (seigniorage). Only after 1982, budget deficits were increasingly financed in a more orthodox way using market mechanisms, through the issuing of public debt, which allowed the development of the secondary market for public debt and the use of central government bonds as reference of the long-term interest rates.

Summing up, the results supports only a "weak" version of the expectations hypothesis of the term structure of interest rates for the Spanish economy. Our

empirical results support a long-run relationship between the long and short interest rates, but this cointegration relationship is not stable. Moreover, the estimate of long-run elasticity is significantly lower than one (in the full sample and second regime), so that changes in the long-term interest rate would have not been fully adjusted to compensate the behaviour of the short-term interest rates.

## References

- [1] Andrews, D.W.K. (1993): "Tests for parameter instability and structural change with unknown change point", *Econometrica*, 61, 821-56.
- [2] Arai, Y. and Kurozoumi, E. (2007): "Testing for the null hypothesis of cointegration with a structural break", *Econometric Reviews*, 26, 705-739.
- [3] Arghyrou, M.G. and Luintel, K.B. (2007): "Government solvency: Revisiting some EMU countries", *Journal of Macroeconomics*, 29, 387-410.
- [4] Bai, J. and Perron, P. (1998): "Estimating and Testing Linear Models with Multiple Structural Changes", *Econometrica*, 66, 47-78
- [5] Bai, J. and Perron, P. (2003): "Computation and analysis of multiple structural change models", *Journal of Applied Econometrics*, 18, 1-22.
- [6] Bank of Spain (2010): *Statistical Bulletin*, March, historical series on internet.
- [7] Camarero, M. and Tamarit, C. (2002): "Instability tests in cointegration relationships. An application to the term structure of interest rates", *Economic Modelling*, 19, 783-799.
- [8] Campbell, J. Y. (1995): "Some lessons from the yield curve", *Journal of Economic Perspectives*, 9, 129-52.
- [9] Campbell, J.Y. and Shiller, R. (1987): "Cointegration and tests of present value models", *Journal of Political Economy*, 95, 1062-1088.
- [10] Campbell, J. Y. and Shiller, R. J. (1991): "Yield spreads and interest rate movements: a birds eye view", *Review of Economic Studies*, 58, 495-514.
- [11] Carrion-i-Silvestre, J.L. and Sansó-i-Roselló, A.S. (2006): "Testing the Null Hypothesis of Cointegration with Structural Breaks", *Oxford Bulletin of Economics and Statistics*, 68, 623-646.
- [12] Cuthbertson, K. (1996): "The expectations hypothesis of the term structure: the UK interbank market", *Economic Journal*, 106, 578-592.
- [13] DeJong, D.N.J., J.C. Nankervis, N.E. Savin and C.H. Whiteman (1992): "Integration versus trend stationary in time series", *Econometrica*, 60, 423-433.

- [14] Elliot, G., T.J. Rothenberg and J.H. Stock (1996): "Efficient test for an autoregressive unit root", *Econometrica*, 64, 813-836.5-465.
- [15] Engsted, T. and Tanggaard, C. (1994a): "A cointegration analysis of Danish zero-coupon bond yields", *Applied Financial Economics*, 24, 265-278.
- [16] Engsted, T. and Tanggaard, C. (1994b): "Cointegration and the US term structure", *Journal of Banking and Finance*, 18, 167-181.
- [17] Esteve, V. (2006): "A note on nonlinear dynamics in the Spanish term structure of interest rates", *International Review of Economics and Finance*, 15, 316-323.
- [18] Gregory, A.W. and Hansen, B.E. (1996a): "Residual-Based Tests for Cointegration in Models with Regime Shifts", *Journal of Econometrics*, 70, 99-126.
- [19] Gregory, A.W. and Hansen, B.E. (1996b): "Tests for Cointegration in Models with Regime and Trend Shifts", *Oxford Bulletin of Economics and Statistics*, 58, 555-560.
- [20] Hall, A. D., Anderson, H. M. and Granger, C. W. J. (1992): "A cointegration analysis of Treasury bill yields", *The Review of Economics and Statistics*, 74, 116-126.
- [21] Hansen, B.E. (1992): "Testing for Structural Change in Conditional Models", *Journal of Econometrics*, 97, 93-115.
- [22] Hansen, B.E. (2000): "Sample Splitting and Threshold Estimation", *Econometrica*, 68, 575-603.
- [23] Hansen, B.E. and Seo, B. (2002): "Testing for two-regime threshold cointegration in vector error-correction models", *Journal of Econometrics*, 110, 293-318.
- [24] Hardouvelis, G. A. (1994): "The term structure spread and future changes in long and short rates in the G7 countries: is there a puzzle?", *Journal of Monetary Economics*, 33, 255-283.
- [25] Jondeau, E. and Ricart, R. (1999): "The expectations hypothesis of the term structure: tests on US, German, French, and UK euro-rates", *Journal of International Money and Finance*, 18, 725-750.
- [26] Kejriwal, M. (2008): "Cointegration with structural breaks: an application to the Feldstein-Horioka Puzzle", *Studies in Nonlinear Dynamics & Econometrics*, 12 (1), 1-37.
- [27] Kejriwal, M. and Perron, P. (2008): "The limit distribution of the estimates in cointegrated regression models with multiple structural changes", *Journal of Econometrics*, 146, 59-73.

- [28] Kejriwal, M. and Perron, P. (2010): "Testing for multiple structural changes in cointegrated regression models", *Journal of Business and Economic Statistics*, 28, 503-522.
- [29] Lanne, M. (2000): "Near unit roots, cointegration, and the term structure of interest rates", *Journal of Applied Econometrics*, 15, 513-529.
- [30] Newey, W. K. and West, K. D. (1987): "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix", *Econometrica*, 55, 703-708.
- [31] Ng, S. and P. Perron (1995), "Unit Root Tests in ARMA Models with Data Dependent Methods for the Selection of the Truncation Lag", *Journal of the American Statistical Association*, 90, 268-281.
- [32] Ng, S. and Perron, P. (2001): "Lag length selection and the construction of unit root tests with good size and power", *Econometrica*, 69, 1529-1554.
- [33] Perron, P. (2006): "Dealing with Structural Breaks", in Palgrave Handbook of Econometrics, Vol. 1: Econometric Theory, K. Patterson and T.C. Mills (eds.), Palgrave Macmillan, 278-352.
- [34] Perron, P. (1997): "Further evidence on breaking trend functions in macroeconomic variables", *Journal of Econometrics*, 80, 355-385.
- [35] Perron, P. (2008): "Structural Change", in The New Palgrave Dictionary of Economics, 2nd ed, S. Durlauf and L. Blume (eds.), Palgrave Macmillan.
- [36] Perron, P. and Ng, S. (1996): "Useful modifications to some unit root tests with dependent errors and their local asymptotic properties", *Review of Economic Studies*, 63, 435-463.
- [37] Perron, P. and Rodriguez, G. (2003): "GLS detrending, efficient unit root tests and structural change", *Journal of Econometrics*, 115, 1-27.
- [38] Phillips, P.C.B. and Hansen, B.E. (1990): "Statistical Inference in Instrumental Variable Regression with I(1) Processes", *Review of Economic Studies*, 57, 99-125.
- [39] Phillips, P.C.B. and Perron, P. (1988): "Testing for a Unit Root in Time Series Regression", *Biometrika*, 75, 335-346.
- [40] Quintos, C.E. and Phillips, P.C.B. (1993): "Parameter Constancy in Cointegrated Regressions", *Empirical Economics*, 18, 675-706.
- [41] Saikkonen, P. (1991): "Asymptotically efficient estimation of cointegration regressions", *Econometric Theory*, 7, 1-21.
- [42] Sarno, L., Thornton, D. L. and Valente, G. (2007): "The empirical failure of the expectations hypothesis of the term structure of bond yields", *Journal of Financial and Quantitative Analysis*, 42, 81-100.

- [43] Shin, Y. (1994): "A residual-based test of the null of cointegration against the alternative of no cointegration", *Econometric Theory*, 10, 91-115.
- [44] Stock, J.H. (1999): "A Class of Tests for Integration and Cointegration", in R.F. Engle and H. White (Eds.): *Cointegration, Causality and Forecasting. A Festschrift in Honour of Clive W.J. Granger*, Oxford University Press, pp. 37-167.
- [45] Stock, J. H. and Watson, M. W. (1988): "Testing for common trends", *Journal of the American Statistical Association*, 83, 1097-1107.
- [46] Stock, J. H. and Watson, M. W. (1993): "A simple estimator of cointegrating vectors in higher order integrated systems", *Econometrica*, 61, 783-820.
- [47] Schwert, G.W. (1989): "Tests for unit roots: A Monte Carlo investigation", *Journal of Business and Economic Statistics*, 7, 147-159.
- [48] Thornton, D. L. (2006) Tests of the expectations hypothesis: resolving the Campbell-Shiller paradox", *Journal of Money, Credit, and Banking*, 38, 511-42.
- [49] Tzavalis, E. (2003): "The term premium and the puzzles of the expectations hypothesis of the term structure", *Economic Modelling*, 21, 73-93.
- [50] Zivot, E. and Andrews, D.W.K. (1992): "Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis", *Journal of Business and Economic Statistics*, 10, 251-270.



Table 1  
Ng-Perron tests for a unit roots

I(2) vs. I(1)		Case: $p = 0, \bar{c} = -7.0$		
<i>Variable</i>	$\bar{M}Z_{\alpha}^{GLS}$	$\bar{M}Z_t^{GLS}$	$\bar{M}SB^{GLS}$	$ADF^{GLS}$
$\Delta bonds_t$	-124.1***	-7.87***	0.063	-8.96***
$\Delta cmr_t$	-181.3****	-9.51***	0.052	-11.65***
I(1) vs. I(0)		Case: $p = 1, \bar{c} = -13.5$		
<i>Variable</i>	$\bar{M}Z_{\alpha}^{GLS}$	$\bar{M}Z_t^{GLS}$	$\bar{M}SB^{GLS}$	$ADF^{GLS}$
$bonds_t$	-3.32	-1.26	0.380***	-1.26
$cmr_t$	-28.15***	-3.73***	0.132	-3.77***

Notes:

<sup>a</sup> A \*, \*\* and \*\*\* denote significance at the 10%, 5% and 1% levels, respectively.

<sup>b</sup> The *MAIC* information criteria is used to select the autoregressive truncation lag,  $k$ , as proposed in Perron and Ng (1996). The critical values are taken from Ng and Perron (2001), table 1:

Critical values:	Case: $p = 0, \bar{c} = -7.0$			Case: $p = 1, \bar{c} = -13.5$		
	10%	5%	1%	10%	5%	1%
$\bar{M}Z_{\alpha}^{GLS}$	-5.7	-8.1	-13.8	-14.2	-17.3	-23.8
$\bar{M}SB^{GLS}$	0.275	0.233	0.174	0.185	0.168	0.143
$\bar{M}Z_t^{GLS}, ADF^{GLS}$	-1.62	-1.98	-2.58	-2.62	-2.91	-3.42

Table 2  
Perron and Rodríguez<sup>a,b</sup> tests for a unit root  
with one time change in the trend function choosing  
the break point minimizing the tests

I(1) vs. I(0)		Case: $p = 1, \bar{c} = -22.5$								
Variable	$M_{MAIC}^{GLS}$ tests						$ADF^{GLS}$	$k$	$T_B$	$\hat{\alpha}$
	$MZ_{\alpha}^{GLS}$	$k$	$T_B$	$MZ_t^{GLS}$	$k$	$T_B$				
$cmr_t$	-26.6*	16	1980:2	-3.64*	16	1980:2	-3.42	16	1980:2	0.91

Notes:

<sup>a</sup> A \*, \*\* and \*\*\* denote significance at the 10%, 5% and 1% levels, respectively.

<sup>b</sup> The *MAIC* information criteria is used to select the autoregressive truncation lag,  $k$ , as proposed Perron and Rodriguez (2003). We impose a minimal value  $k = 1$ . The critical values are taken from Perron and Rodriguez (2003), table 1 (a), Model II,  $T = 200$ :

Critical values:	Case: $p = 1, \bar{c} = -22.5$		
	10%	5%	1%
$\bar{M}Z_{\alpha}^{GLS}$	-23.7	-27.1	-33.5
$\bar{M}Z_t^{GLS}$	-3.42	-3.63	-4.07
$ADF^{GLS}$	-3.62	-3.90	-4.48

Table 3

Perron and Rodríguez<sup>a,b</sup> tests for a unit root with one time change in the trend function choosing the break point maximizing  $|t_{\hat{\beta}_2}|$

I(1) vs. I(0)	Case: $p = 1, \bar{c} = -22.5$					
Variable	$M_{MAIC}^{GLS}$ tests			$k$	$T_B$	$\hat{\alpha}$
	$\bar{M}Z_{\alpha}^{GLS}$	$\bar{M}Z_t^{GLS}$	$ADF^{GLS}$			
$cmr_t$	-15.9	-2.82	-3.06	16	1979:9	0.93

Notes:

<sup>a</sup> A \*, \*\* and \*\*\* denote significance at the 10%, 5% and 1% levels, respectively.

<sup>b</sup> The *MAIC* information criteria is used to select the autoregressive truncation lag,  $k$ , as proposed Perron and Rodriguez (2003). We impose a minimal value  $k = 1$ . The critical values are taken from Perron and Rodriguez (2003), table 1 (b), Model II,  $T = 200$ :

Critical values:	Case: $p = 1, \bar{c} = -22.5$		
	10%	5%	1%
$\bar{M}Z_{\alpha}^{GLS}$	-21.4	-24.5	-31.2
$\bar{M}Z_t^{GLS}$	-3.24	-3.47	-3.91
$ADF^{GLS}$	-3.42	-3.67	-4.25

Table 4  
 Estimation of long-run relationships: Stock-Watson-Shin cointegration tests

Parameter estimates	Full sample 1974:1-2010:2	First regime 1974:1-1979:6	Second regime 1979:7-2010:2
$c$	2.20 (2.88)	10.15 (7.51)	1.86 (4.15)
$\gamma$	0.77 (11.4)	0.09 (1.04)	0.83 (20.1)
$R^2$	0.98	0.99	0.99
$C_\mu$	0.114	0.131	0.087
$W_{DOLS}$	12.53*	—	18.12*

Notes:

<sup>a</sup>  $t$ -statistics are in brackets. Standard Errors are adjusted for long-run variance. The long-run variance of the cointegrating regression residual is estimated using the Barlett window which is approximately equal to  $INT(T^{1/2})$  as proposed in Newey and West (1987).

<sup>b</sup> We choose  $q = INT(T^{1/3})$  as proposed in Stock and Watson (1993).

<sup>c</sup>  $C_\mu$  and  $C_\tau$  are  $LM$  statistics for cointegration using the DOLS residuals from deterministic and stochastic cointegration, respectively, as proposed in Shin (1994). A \*, \*\* and \*\*\* denote significance at the 10%, 5% and 1% levels, respectively.

<sup>d</sup> The critical value for a  $\chi_1^2$  at 5%: 3.84.

<sup>e</sup> The critical values are taken from Shin (1994), table 1, from  $m = 1$ :

Critical values:			
	10%	5%	1%
$C_\mu$	0.231	0.314	0.533

Table 5  
Kejriwal-Perron tests for testing multiple structural breaks  
in cointegrated regression models: equation (5) and (11)<sup>a,b</sup>

Specifications			
$y_t = \{bonds_t\}$	$z_t = \{1, cmr_t\}$		
$q = 2$	$p = 0$	$h = 64$	$M = 5$
Test			
$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$\sup F_T(4)$
12.58**	8.41	6.39	5.02
$\sup F_T(5)$	$WD \max$		
4.17	12.58**		
Number of Breaks	Break dates		
Selected	estimates <sup>c</sup>		
1	$\hat{T}_1: 1979:6 [1979:4-1980:11]$		

Notes:

<sup>a</sup>  $y_t$ ,  $z_t$ ,  $q$ ,  $p$ ,  $h$ , and  $M$  denote the dependent variable, the regressors, the number of I(1) variables (and the intercept) allowed to change across regimes, the number of I(0) variables, the minimum number of observations in each segment, and the maximum number of breaks, respectively.

<sup>b</sup> \*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels, respectively. The critical values are taken from Kejriwal and Perron (2010), Table 1, nontrending case,  $q_b = 1$ .

<sup>c</sup> In parentheses, reported are the 95% confidence intervals for the break dates.

Table 6  
Arai-Kurozumi-Kejriwal cointegration tests with a single structural  
break: equation (5) and (11)

Test	$\tilde{V}_k(\hat{\lambda})^a$	$\hat{\lambda}_1$	$\hat{T}_1$
	0.147	0.15	1979:6

Notes:

<sup>a</sup> \*, \*\*, and \*\*\* denote significance at the 10%, 5%, and 1% levels, respectively.

<sup>b</sup> Critical values are obtained by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.*  $N(0, 1)$  random variables:

Critical values:	10%	5%	1%
$\tilde{V}_k(\hat{\lambda})$	0.168	0.223	0.425

Figure 1  
Spanish nominal short-term and long-term interest rates  
1974:01-2010:02

