Historical financial analogies of the current crisis

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Abstract

This paper tries to shed light on the historical analogies of the current crisis. To that end we compare the current sample distribution of Dow Jones Industrial Average Index returns for a 769-day period (from 15 September 2008, the Lehman Brothers bankruptcy, to September 2011), with all historical sample distributions of returns computed with a moving window of 769 days in the 2 January 1900 to 12 September 2008 period. Using a Kolmogorov-Smirnov and a $\chi^2$ homogeneity tests which have the null hypothesis of equal distribution we find that the stock market returns distribution during the current crisis would be similar to several past periods of severe financial crises that evolved into intense recessions, being the sub-sample from 28 May 1935 to 17 Jun 1938 the most analogous episode to the current situation. Furthermore, when applying the procedure proposed by Diebold, Gunther and Tay (1998) for comparing densities of sub-samples, we obtain additional support for our findings and discover a period from 10 September 1930 to 13 October 1933 where the severity of the crisis overcomes the current situation having sharper tail events. Finally, when comparing historical market risk with the current risk, we observe that the current market risk has only been exceeded at the beginning of the Great Depression.

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1. Introduction.

There is a burgeoning literature on determining the causes of the current global crisis and on finding precursors in past global crises (see, e. g. Reinhart and Rogoff, 2009a). In contrast with the main avenue of research in this literature that, following Eichengreen et al. (1995), Kaminsky and Reinhart (1999) and several subsequent authors, examines the behaviour of key economic variables around crisis episodes, this paper tries to shed light on the historical analogies of the current crisis making use of a battery of statistical tests to detect past sub-periods where the distribution of the Dow Jones Industrial Average Index returns are similar to the most recent sub-sample covering the current crisis outbreak.

The reason for studying the distribution of returns for the stock market in the United States is given by the fact that, while the crisis initially had its origin in this country in a relatively small segment of the lending market (the sub-prime mortgage market), it rapidly spread across virtually all economies, affecting stock markets worldwide, and so, many countries experienced even sharper stock market crashes than the United States. Moreover, starting with Fisher (1933), a number of researchers emphasize the importance of financial cycles for the real economy and there are many studies indicating that stock returns are related to current and future levels of economic activity (see, e. g., Grossman and Shiller, 1981).

The paper is organised as follows. Section 2 presents the econometric methodology. Section 3 describes the data set and reports the empirical results. Finally, Section 4 offers some concluding remarks.

2. Econometric methodology.

We detect analogies to the current crisis using a Kolmogorov–Smirnov test (KS test). This is a nonparametric test for the equality of continuous, one-dimensional probability distributions that can be used to compare two sub-samples (see Rohatgi, 1976). The null distribution of this statistic is calculated under the null hypothesis that the samples are drawn from the same distribution (i. e., equal distributions for both sub-samples), and the alternative corresponds to different distributions.
Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be independent random samples of returns having unknown continuous distribution functions $F(x)$ and $G(x)$ respectively.

In order to establish the hypothesis test:

$$H_0 : \quad F(x) = G(x) \quad \text{for} \quad -\infty < x < \infty \quad (1)$$

$$H_1 : \quad F(x) \neq G(x) ,$$

we consider the sample distribution functions defined as

$$F_n^*(x) = \sum_{j=1}^{n} \varepsilon(x - X_j) \quad \text{and} \quad G_m^*(x) = \sum_{j=1}^{m} \varepsilon(x - Y_j)$$

where the function $\varepsilon(.)$ is the function

$$\varepsilon(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

so, $nF_n^*(x)(mG_m^*(x))$ is the number of $X_j(Y_j)$ that are $\leq x$.

Let us consider the statistic

$$D_{n,m} = \sup_{-\infty < x < \infty} |F_n^*(x) - G_m^*(x)|$$

if the null $H_0$ is true the statistic $\left( \frac{mn}{m+n} \right)^{1/2} D_{n,m}$ converges in distribution to an expression given by the following numerical series:

$$\lim_{n \to \infty} P \left( \left( \frac{mn}{m+n} \right)^{1/2} D_{n,m} \leq t \right) = 1 - 2 \sum_{i=1}^{\infty} (-1)^{i-1} e^{-2i^2}$$

This result can be used to provide the critical value $d_\alpha$ for accepting the null $H_0$, that is to find $d_\alpha$ such that

$$\lim_{n \to \infty} P \left( \left( \frac{mn}{m+n} \right)^{1/2} D_{n,m} \leq d_\alpha \right) = 1 - \alpha ,$$

and tables of $d_\alpha$ for various values of $\alpha$ are available in Owen (1962). For instance, it is easy to check using (2) that...
\[ P \left( n \left( \frac{mn}{m+n} \right)^{1/2} D_{n,m} \leq 1.36 \right) = 0.95 \] and so, we can reject the null \( H_0 \) when
\[ \left( \frac{mn}{m+n} \right)^{1/2} D_{n,m} > 1.36 \] with a significance level of 0.05.

As additional evidence for detecting analogies to the current crisis we have also considered the Chi-Square Test for Homogeneity (see Rohatgi, 1976). Now let \( A_1, A_2, \ldots, A_k \) be a partition of the real line, and let \( X_1, X_2, \ldots, X_n \) and \( Y_1, Y_2, \ldots, Y_m \) be independent random samples of returns, like above. In what follows \( n=m \) for convenience. Let \( X(i) \) be the number of observations in \( X_1, X_2, \ldots, X_n \) that lie in the set \( A_i \), and \( Y(i) \) be the number of observations in \( Y_1, Y_2, \ldots, Y_m \) that lie in the set \( A_i \). Let
\[ \hat{p}_i = \frac{X(i)}{2n} + \frac{Y(i)}{2n}, \quad i=1,2,\ldots,k. \]

In this case, the random variable
\[ \sum_{i=1}^{k} \frac{(X(i) - n\hat{p}_i)^2}{n\hat{p}_i} + \sum_{i=1}^{k} \frac{(Y(i) - n\hat{p}_i)^2}{n\hat{p}_i} \] is approximately \( \chi^2 \), with \( k-1 \) degrees of freedom, which permits us to establish a second version of the hypothesis test (1).

Some authors cast doubts on the practical applications of the KS and \( \chi^2 \) tests because they are not constructive and, so, when rejection of \( H_0 \) occurs, the tests generally provide no guidance as to why: because the samples are not independent, because the samples have different distributions or both. In this sense Diebold, Gunther and Tay (1998) (DGT hereafter) have provided a new test for comparing densities of sub-samples. Given the sample density function \( p_X(u) \) of a sample \( X_1, X_2, \ldots, X_n \), the probability integral transformation of another sample \( Y_1, Y_2, \ldots, Y_m \) is the cumulative density function corresponding to the density \( p_X(u) \) evaluated at \( Y_i \),
\[ Z_i = \int_{-\infty}^{Y_i} p_X(u) \, du \]
Under the null hypothesis that \( X_1, X_2, \ldots, X_n \) and \( Y_1, Y_2, \ldots, Y_m \) are independent random samples having a common unknown continuous distribution, the \( Z_1, Z_2, \ldots, Z_m \) must be independent and uniformly distributed \( U(0,1) \) in the interval \((0,1)\).

DGT (1998) propose a graphical procedure for rejecting the null based on looking at the histogram of the probability integral transformation. This procedure consists of comparing the estimated density of the probability integral transformation (3) to a \( U(0,1) \) by computing confidence intervals under the null hypothesis of i.i.d. \( U(0,1) \).

Besides, in order to evaluate whether \( Z_t \) in (4) is i.i.d., they propose using the correlogram, supplemented with the usual Bartlett confidence intervals. In this sense, serial correlation in the \( Z_t - \bar{Z}_t \) series indicates that the conditional mean dynamic of the returns \( X_t \) are different to the conditional mean dynamic of the returns \( Y_t \). If potentially sophisticated nonlinear forms of dependence are looked for, it is necessary examine the correlograms of powers of \( Z_t - \bar{Z}_t \), that is \((Z_t - \bar{Z}_t)^2\), \((Z_t - \bar{Z}_t)^3\) and \((Z_t - \bar{Z}_t)^4\).

3. Data and empirical results.

3.1 Data.

In this paper we use daily data of the Dow Jones Industrial Average Index (DJIA) from 2 January 1900 to 30 September 2011 provided by Reuters’ EcoWin Pro\(^1\). We first compute daily returns for this period and calculate the histogram of all probability distributions obtained using a moving 769-day window. We then make use of the KS test to compare all these histograms with the histogram computed for the last 769 days in the sample, covering the period from the bankruptcy of Lehman Brothers Holdings Inc. on 15 September 2008 to the end of the sample. We take the collapse of Lehman Brothers Holdings Inc. as a breaking point, since it is thought to have played a major role in the unfolding of the current global financial crisis and in the abrupt contraction of economic activity registered worldwide.

\(^1\) The DJIA and Standard and Poor’s 500 Composite (S&P500) indexes are very highly correlated with each other, telling a similar story in levels, returns and volatility. The use of DJIA is likely to be sufficient for analysing the issues at hand.
3.2 The KS and $\chi^2$ tests

Figure 1 plots the computed values of the KS statistic when comparing all the histograms of possible successive 769-day returns computed for the DJIA from 2 January 1900 with the histogram associated with 769-day returns after the bankruptcy of Lehman Brothers. The dashed line corresponds to the critical value of 1.36. As stated before, computed tests greater than 1.36 reject the null hypothesis of equal distributions of both sub-samples at a significance level of 95%. The minimum value is obtained when comparing with the subsample starting on 28 May 1935 and the maximum value is reached when comparing with the subsample starting on 10 September 1930.

Figure 1: Historical evolution of Kolmogorov–Smirnov test comparing the DJIA returns in the current crisis to past periods.

As is shown in Figure 1, the past periods where the KS test does not reject the null hypothesis of equal distribution of stock returns to the last 769-day sub-sample are the following:

- **I**: 28 August 1905 to 25 October 1909. This sub-sample covers the Panic of 1907, a financial crisis caused by a retraction of market liquidity by a number of New York City banks that evolved to economic recession, with numerous runs on banks and trust companies.

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2 For the history of financial crises, see Kindleberger and Aliber (2005) and Reinhart and Rogoff (2009).
• **II**: 11 April 1914 to 26 July 1922. This sub-sample includes the short but extremely painful recession of 1920–1921.

• **III**: 26 January 1927 to 16 December 1930. This sub-sample embraces a period of stock markets crashes worldwide leading to the Great Depression.

• **IV**: 30 December 1932 to 11 August 1941. This sub-sample encompasses the recession of 1937, which is among the worst recessions of the 20th century.


• **VI**: 6 February 1986 to 14 April 1989. This sub-sample covers the 1987 stock market crash.

• **VII**: 22 July 1987 to 21 November 1990. This sub-sample includes the 1990 oil price shock and the early 1990s recession.


• **IX**: 12 January 2001 to 4 August 2005. This sub-sample encompasses the WorldCom bankruptcy in 2002 (the largest in the history of the United States at the time) and the 2003 turbulence in stock markets related to a pessimistic outlook for the global economy and increased uncertainty.

As additional evidence of historical analogies with the current crisis we have also considered the $\chi^2$ homogeneity test. Figure 2 plots the computed values of the $\chi^2$ statistic in (3) comparing the returns of the current crisis to past 769 day episodes during the history of DJIA. The dashed line corresponds to the critical value of $\chi^2_{k-1} = 30.14$ at a significance level of 95%. As can be observed, the shape of the curves in Figure 1 and Figure 2 is very similar. The main difference is that, in the $\chi^2$ test, the null of equal distribution is only accepted in period III and period IV of Figure 1.
Figure 2: Historical evolution of $\chi^2$ test comparing the DJIA returns in the current crisis to past periods.

Therefore, the KS and $\chi^2$ tests reveal analogies between the current situation and past economic recessions, suggesting that the world economy could be heading towards a new and marked slowdown if it evolves as in similar situations that in the past.

Given that the KS and $\chi^2$ tests take the minimum value when comparing the current crisis with the sub-sample running from 30 December 1932 to 11 August 1941, it seems that, if history repeats itself, current high uncertainty and intensified downside risks could lead to a higher probability of a double-dip recession. Indeed, from 1933 to 1936, the US economy grew vigorously, output nearly returning to its level of 1929. But in 1937, the recovery halted and the economy fell back into a second recession. This would be in line with the conclusion of a sizable body of empirical literature stating that recessions caused by financial crises have a history of being long, deep and difficult to fully escape (e.g., Reinhart and Rogoff, 2009b).

Our results are also consistent with Baur, Quintero and Stevens (1996) who report that during the periods that surrounded the crash, only changes in fundamentals have a statistically significant impact on the movement of stock prices, as well as with Shachmurove (2011) who, after examining the economic history of the United States, concludes that financial crises and banking panics are not exclusive of the nineteenth-century, but that these phenomena are still reoccurring.
3.3 The DGT procedure.

As further evaluation of the analogies detected in the KS and $\chi^2$ tests between the historical and present return, we have applied the graphic framework developed by DGT (1998) to the periods detected in Figure 1 and Figure 2 where the KS and $\chi^2$ statistics take an extreme value (a local minimum or a local maximum). In all cases, the density function $p_x(u)$ in the expression (4) was the empirical distribution of the 769-day sub-sample starting on 15 September 2008 (the bankruptcy of Lehman Brothers), and the random variable $Y_t$ in (4) was the historical period returns which we want to compare. As it will be shown, the nature of these histograms provided by the DGT test are completely different.

For the case of local minima of the KS and $\chi^2$ statistics in Figure 1 and Figure 2, where the null hypothesis of similar return distribution with the current crisis is accepted, the shape of the histograms for the probability integral transformation correspond to a uniform distribution, as could be expected. As an example of this behaviour, Panel A in Figure 3 shows the histogram of the probability integral transformation corresponding to the sub-sample beginning on 28 May 1935, where the KS and $\chi^2$ take the absolute minimum, the dashed lines being the binomial confidence bands for a confidence level of 99%. So, this histogram corresponds to a $U(0,1)$ variable. It suggests that the empirical density $p_x(u)$ (corresponding to the last subsample running from 15 September 2008 to 30 September 2011) and the density associated with the period covering from 28 May 1935 to 17 June 1938 have similar properties. The histogram obtained using the DGT procedure is also close to the uniform in the rest of local minima of KS and $\chi^2$ statistics in Figure 1 and Figure 2.

Now let us consider the histogram where the null is strongly rejected. Neither the KS nor the $\chi^2$ tests specify the reason for rejection, however the DGT test does, and two patterns emerge in the histograms rejecting the uniformity. On the one hand, the KS and $\chi^2$ rejection of similarity with the current crisis could be produced do to the fact that the returns in the analysed period have a lower volatility, the tail events are less frequent.

3 Monte Carlo simulations show that the null can also be accepted with a confidence level of 95%.
4 Observe that our sample size is 769 whereas in DGT (1998) the graphical exercise of comparing histograms was carried out with a sample size of 4000 observations.
and the market risk is lower. For instance, this is the case represented by Panel B in Figure 3 which corresponds to the histogram associated with the period from 7 January 1963 to 24 January 1966, where KS and $\chi^2$ tests have rejected an equal distribution compared to the current crisis. In this case the histogram has a non-uniform inverted U shape, suggesting that the empirical density $p_u(u)$ (corresponding to the last subsample running from 15 September 2008 to 10 September 2011) has a different density than the sub-sample (taken from 7 January 1963 to 24 January 1966) since both empirical densities have completely different tails. So, in (4) $X_1, X_2, \ldots, X_n$ present extreme values with respect to $Y_1, Y_2, \ldots, Y_n$.

The pattern shown in the histogram in Panel B of Figure 3 is also present in all the local maxima reached by KS and $\chi^2$ statistics with one exception. This exception corresponds to the absolute maximum of these statistics in Figure 1 and Figure 2, and it relates to the period from 10 September 1930 to 13 October 1933, when the financial crisis was extremely severe and major bank panics occurred (Friedman and Schwartz, 1963). Panel C of Figure 3 shows the histogram corresponding to the DGT test for this period. The U shape of the histogram suggests that the returns in this period have a higher volatility, the tail events are more frequent, and the market risk is higher than in the current crisis. In terms of the empirical density $p_x(u)$ (corresponding to the current crisis), it means that the period taken from 10 September 1930 to 13 October 1933, has a different density, that is $Y_1, Y_2, \ldots, Y_n$ present extreme values with respect to $X_1, X_2, \ldots, X_n$ in (4). The U shape in the histograms of the probability integral transformation is also found in the sub-periods beginning around the absolute maximum of the KS and $\chi^2$ statistics corresponding to Figure 1 and Figure 2.
Figure 3: Histograms of Diebold et al. (1998) test with binominal confidence bands

Notes:

- Panel A corresponds to the comparison of the current crisis with the KS and $\chi^2$ absolute minimum, where the similarity is accepted.
- Panel B corresponds to the comparison of the current crisis with several local maxima of KS and $\chi^2$ statistics. Here the current crisis has fatter tails.
- Panel C corresponds to the comparison of the current crisis with the absolute maximum of KS and $\chi^2$ statistics. The current crisis is less severe.

Following the DGT methodology, it is possible to evaluate whether $Z_t$ in (4) is i.i.d., looking for a serial correlation in the $Z_t - \overline{Z}_t$ series which indicates that the conditional mean dynamic of the returns $X_t$ (corresponding to the 769-day period after the Lehman Brothers default) are different to conditional mean dynamic of the returns $Y_t$ (corresponding to the 769-day period from 28 May 1935 to 17 Jun 1938 where the KS and $\chi^2$ statistics take the minimum value in Figure 1 and Figure 2). Moreover, potentially sophisticated nonlinear forms of dependence may be looked at for examining the correlograms of powers of $Z_t - \overline{Z}_t$. 
In Figure 4 we show the sample autocorrelations of $Z_t - \bar{Z}_t$, $(Z_t - \bar{Z}_t)^2$, $(Z_t - \bar{Z}_t)^3$ and $(Z_t - \bar{Z}_t)^4$ and the critical values $\pm 2/\sqrt{T}$ (where $T$ is the sample size) of the test $H_0: \rho = 0$. As can be observed in Figure 4, the correlograms show no evidence of neglected dynamics of $Y_t$ returns series with respect to $X_t$ series.

**Figure 4: Correlograms of**

$Z_t - \bar{Z}_t$, $(Z_t - \bar{Z}_t)^2$, $(Z_t - \bar{Z}_t)^3$ **and** $(Z_t - \bar{Z}_t)^4$

Notes:

(a) $Z_t - \bar{Z}_t$.
(b) $(Z_t - \bar{Z}_t)^2$.
(c) $(Z_t - \bar{Z}_t)^3$.
(d) $(Z_t - \bar{Z}_t)^4$.

The correlogram of $Z_t - \bar{Z}_t$ and their powers reveals that, although significant serial correlation in the series doesn’t exist, nonlinear dependences exist between $Z_t$ and $Z_{t-h}$. The strong serial correlation in $(Z_t - \bar{Z}_t)^2$ and in $(Z_t - \bar{Z}_t)^4$ reveals operative dependence
through conditional variance and conditional kurtosis. So, the sample after the Lehman Brothers default and the sample where the KS and $\chi^2$ statistics take a minimum value present different behaviour from the dynamical point of view of the conditional variance, even though the histogram in the DGT procedure (Panel A Figure 2) does not reject the null hypothesis of equal distributions.

As additional evidence of the rejection of the null hypothesis for equal sample distribution, we have also studied the historical acceptance of similarity by the DGT test comparing the current crisis with past episodes. In this case the 95% critical values in the DGT test were obtained by Monte Carlo simulations and a total of 1480 acceptances of a similar distribution, out of 27796 769-day periods considered in the history of the DJIA, have been supplied (that is, 0.0532% of times). The acceptances of the null in DGT are produced around the local minimum values of KS and $\chi^2$ statistics, especially during the period IV where their absolute minimum was found. The results are displayed in Figure 5 and the vertical lines show the 769-day sample periods where the null hypothesis of equal distribution was accepted.

**Figure 5: Historical acceptance of similarity using the Diebold et al. (1998) test comparing the current crisis with past episodes.**

Therefore, the DGT methodology provides deeper insight into our earlier conclusion from the KS and $\chi^2$ tests.
3.4 Market risk evolution.

Finally, we have also compared the historical evolution of market risk with its current level during the financial crisis. A well-known measure of risk used in finance is the Value at Risk (VaR). For a given portfolio and time horizon, $VaR_\alpha$ is defined as a threshold value such that the probability that the loss on the portfolio exceeds this value is the given probability level $1 - \alpha$. Nevertheless, the usefulness of VaR as a measure of risk is highly questionable outside the confines of near-normal distributions and one important limitation is that VaR only tells us the most we can lose if a tail event does not occur (e.g., it tells us the most we can lose 95% of the time); if a tail event does occur, we can expect to lose more than VaR, and the VaR itself gives us no indication of how much that might be.

An alternative risk measurement to VaR frequently employed in empirical applications, is the conditional VaR (CVaR), also known as expected shortfall or tail-VaR (see Artzner et al., 1999). This risk assessment technique is more sensitive to the shape of the loss distribution in the tail, and is performed by assessing the likelihood (at a specific confidence level, $\alpha$) that a specific loss will exceed the value at risk, being a more consistent measure of risk compared to VaR since it is sub-additive and convex.

The CVaR is the expected value of the losses exceeding the VaR, that is

$$CVaR_\alpha = E[L / L > VaR_\alpha]$$

Therefore, it is a weighted average of losses for the worst $100(1 - \alpha)$% of cases exceeding VaR with a confidence level $\alpha$.

In order to estimate the CVaR from our empirical distributions of returns and following Dowd (2005), we slice the tail into a large number $n$ of slices, each of which has the same probability mass, estimate the VaR associated with each slice, and take the CVaR as the average of these VaRs.

In Figure 6 we show the historical behaviour of one-day 95% CVaR estimated averaging 50 VaRs with confidence level from 95.1% to 99.9%. The horizontal dashed line represents the CVaR corresponding to the 769-day period after the Lehman
Brothers default. As can be seen in Figure 6, the current market risk assessment has only been exceeded at the beginning of the Great Depression, and the maximum level of CVaR corresponds to the period from 24 October 1929, to 20 October 1932. This period corresponds to the maximum of KS and $\chi^2$ statistics in Figure 1 and Figure 2, where the DGT test produces a U shape histogram (Panel C in Figure 3) revealing tail events deeper than during the current crisis.

Figure 6. Market risk: historical one-day CVaR compared with its current level shown by the dashed line


The current global financial crisis is without precedent in post-war economic history. Although its size and extent are exceptional, the crisis may have features in common with similar financial-stress driven recession episodes in the past.

In this paper we have tried to identify analogies in past experiences with the current financial crisis. To that end, we have first computed returns form the DJIA Index using a moving 769-day window from 2 January 1900 to 30 September 2011 and, applying the Kolmogorov-Smirnov and the $\chi^2$ homogeneity tests, detecting similarities between the histogram associated with the last 796 observations (from the bankruptcy of Lehman
Brothers to the end of the sample) with those corresponding to severe financial crises that evolved into intense recessions. Furthermore, our results also indicate that the most similar episode to the current crisis is the Great Depression of the 1930s, suggesting that the world economy could be entering a new phase of economic weakening, with a high probability of re-entering recession.

To explore the robustness of these results, we have also made use of the graphic method framework proposed by Diebold, Gunther and Tay (1998) for comparing densities of sub-samples, obtaining further support for our findings. Additionally, we have computed the conditional value to compare the historical risk to the current risk, concluding that the current market risk has only been exceeded in a period during the Great Depression.

We believe that our results might have both some practical meaning for investors and policy makers and some theoretical insights for academic scholars interested in business cycles.

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