DOCUMENTOS DE ECONOMÍA Y FINANZAS INTERNACIONALES

Working Papers on International Economics and Finance

DEFI 11-09
November 2011

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Henry Aray

Asociación Española de Economía y Finanzas Internacionales
www.aeefi.com
ISSN: 1696-6376

Hysteresis and Import Penetration with Decreasing Sunk Costs

Henry Aray*
University of Granada

July 8, 2011

Abstract

This article proposes an extension of Dixit (1989, *Quart. J. Econ.*), assuming that potential exporting firms benefit from the experience of firms already settled in the foreign market which allows the sunk cost to diminish. In general, the numerical results show that hysteresis is lower as expected. More interestingly, hysteresis is decreasing with the number of firms. As regards the Dixit case, decreasing sunk cost has a greater impact on entering than on exiting. Finally, the combination of expected depreciation/appreciation rate and sunk cost has striking implications on the import share.

Key words: Real Options, Exchange Rate, Sunk Cost.

JEL Classification: F14, F31, L16.

*Corresponding author: Universidad de Granada. Facultad de Ciencias Económicas y Empresariales. Dpto. de Teoría e Historia Económica. Campus de la Cartuja S/N, 18011 Granada, España. e-mail: haray@ugr.es. Phone: +34958241000 ext. 20140. Fax: +3495849995.

1 Introduction

Dixit (1989) proposes a very interesting model of entry in an foreign market under exchange rate uncertainty. The model considers the option of Japanese firms exporting to a competitive market in the US. Real option theory is used to determine the trigger exchange rates that produce the entry and exit of firms. Firms sequentially enter the US market, with firms whose marginal costs are lower being the first in entering. Undertaking the exporting project implies incurring a sunk cost, which is assumed to be constant for all firms.

Recently, Benkler (2006) established a relationship between the fixed costs of entry into a new market and the degree of familiarity with this market, such that the fixed cost of entry is lower as long as the entrants (potential) are more familiar with the idiosyncrasies of the new markets.

Why can potential entrants get account with new markets? The answer could be given by the literature on urbanization and localization economies which claims that positive externalities can be generated due to the proximity among firms. Desmet and Fafchamps (2005) distinguish between localization economies, which derive from being located close to other firms in the same industry; and urbanization economies, associated with closeness to overall economic activity. Interactions among firms allow the exchange of useful information among their trading partners about the functioning of foreign markets, administrative rules, etc. Hence, export barriers could be reduced if the firms exchange information about the target market. The exchange of information among firms could be made easier by the spacial concentration of firms. Head *et al.* (1995) and Aitken *et al.* (1997) show that the spacial concentration of exporting firms helps other local firms in the internationalization process. The argument is that proximity among firms could increase the transmission of knowledge on the exportation practice and thus facilitate the flows of information about the target country. Becchetti and Rossi (2000) find strong evidence for the positive impact of industrial district on the probability of exporting and the export intensity of small- and medium-sized Italian firms. Similar results were found by Castillo and Requena (2007) for Spain and Koening (2009) for France.

Following the above literature, our assumption is that more information about the foreign market from both the demand and supply side allow the sunk cost to decrease, thus fostering early penetration in the target market. We consider that when there are more exporters in an industry, more information is available for potential exporters. Therefore, we propose a slight extension of Dixit (1989) allowing sunk cost to decrease with the number of exporting firms already settled in the foreign market.

With respect to the Dixit case, our numerical results show that decreasing sunk costs have a greater impact on the option of activating than on the option of abandoning. For any given exchange rate, the lower the sunk cost, the narrower the range of number of firms in equilibrium in the industry.

With regard to this question, one of the main contributions of this article is that unlike the Dixit case where hysteresis increases as long as new firms enter, in industries where the marginal contribution to the reduction of the sunk cost of the new adding firm is constant or increasing, the hysteresis decreases with the number of firms. However, in industries with decreasing marginal contribution to the reduction of the sunk cost of the new adding firm, mixed results are found. For the first third of entering firms, the hysteresis decreases, but increases for the following firms. Finally, regarding market share, our results suggest that countries whose exchange rate suffers from exchange rate depreciation in the long term, the positive growth of the exchange rate hides the positive effects of decreasing sunk costs. In contrast, countries whose exchange rate suffers from exchange rate appreciation in the long term, the negative growth of the exchange rate can be dampened by the positive effects of decreasing sunk costs.

The article is organized as follows. A summary of the Dixit model is presented in section 2. Section 3 provides the numerical results, while conclusions are drawn in section 4.

2 The Dixit's Model

Dixit's model considers that all the uncertainty comes from the exchange rate, R, defined as the number of Japanese currency units (Yen) necessary to buy one unit of US currency (Dollar). R is assumed to follow a geometric

Brownian motion as

$$\frac{dR}{R} = \mu dt + \sigma dz$$

where dz is an increment of a standard Wiener process, is uncorrelated across time and at any one instant satisfies $E(dz) = 0$, $E(dz^2) = dt$, where E denotes the expectations operator. The parameter μ is the expected depreciation rate and σ is a measure of exchange rate volatility.

Dixit (1989) considers N Japanese firms that are potential suppliers of imports to the US. The firms are risk neutral and have rational expectations, that is, they maximize the expected present value of profits in yen. When in the US market, they act as price-takers. Each must incur a sunk capital cost of k dollars to enter this market. This cannot be recouped if the firm should decide to quit at a later date. Moreover, the firm may have to pay a further l dollars to shut down. If it should decide to reenter at a still later date, it must spend k.

Each active firm sells one unit of output per unit of time in the market and acts as a price-taker. The variable cost of supplying the unit of output to the foreign market for the firm n is w_n yen.

The equilibrium of the industry evolves as if to maximize the expected present discounted value of the overall surplus of this industry measured in yen, $V(R)$.

The problem is solved as a sequence of option pricing problems such as

$$\frac{E\left[dV_n\left(R\right)\right]}{dt} + RU\left(n\right) - W\left(n\right) = \rho V_n\left(R\right) \tag{1}$$

Where $U\left(n\right)$ is the utility of the industry when there are n firms and $W\left(n\right)$ is the variable cost of the first n firms

$$\begin{aligned} U\left(n\right) &= \sum_{j=1}^{n} p_j \\ W\left(n\right) &= \sum_{j=1}^{n} w_j \end{aligned}$$

Where p_j is the marginal contribution of firm j to utility and w_j the variable cost of firm j.

Equation (1) states that the capital gains plus the dividend flow must equal the normal return. The general solution to the equation is

$$V_n\left(R\right) = A\left(n\right)R^{-\alpha} + B\left(n\right)R^{\beta} + \frac{RU\left(n\right)}{\rho - \mu} - \frac{W\left(n\right)}{\rho}$$

where $-\alpha < 0$ and $\beta > 1$, and $A\left(n\right)$ and $B\left(n\right)$ are constants to be determined. The boundary conditions for the endpoints $n = 0$ and N, $A\left(0\right) = 0$ and $B\left(N\right) = 0$ are imposed. The interpretation of the conditions are as follows: when there are no established firms, the shutdown option takes a value of zero, and when there is the maximum possible number of firms, the option to add more is worthless.

Let I_n be the exchange rate at which it only becomes optimal to introduce the nth firm. When previously there were $(n-1)$, the value-matching and smooth pasting conditions are

$$
\begin{aligned}
V_{n-1}\left(I_n\right) &= V_n\left(I_n\right) - kI_n \\
V'_{n-1}\left(I_n\right) &= V'_n\left(I_n\right) - k
\end{aligned}
$$

Similarly, let D_n be the exchange rate at which it only becomes optimal that a firm abandons. When there are n firms, the value-matching and smooth pasting conditions are

$$
\begin{aligned}
V_n\left(D_n\right) &= V_{n-1}\left(D_n\right) - lD_n \\
V'_n\left(D_n\right) &= V'_{n-1}\left(D_n\right) - l
\end{aligned}
$$

Considering that

$$
\begin{aligned}
a_n &= A\left(n\right) - A\left(n-1\right) \\
b_n &= B\left(n-1\right) - B\left(n\right)
\end{aligned}
$$

$$
\begin{aligned}
A\left(n\right) &= \sum_{j=1}^{n} a_j \\
B\left(n\right) &= \sum_{j=n+1}^{N} b_j
\end{aligned}
$$

The four-equation system to be solved for a_n,b_n, I_n and D_n can be written as

$$
\begin{aligned}
a_n I_n^{-\alpha} - b_n I_n^{\beta} + \frac{I_n p_n}{\rho - \mu} - \frac{w_n}{\rho} - k I_n &= 0 \\
-\alpha a_n I_n^{-\alpha-1} - \beta b_n I_n^{\beta-1} + \frac{p_n}{\rho - \mu} - k &= 0 \\
a_n D_n^{-\alpha} - b_n D_n^{\beta} + \frac{D_n p_n}{\rho - \mu} - \frac{w_n}{\rho} + l D_n &= 0 \\
-\alpha a_n D_n^{-\alpha-1} - \beta b_n D_n^{\beta-1} + \frac{p_n}{\rho - \mu} + l &= 0
\end{aligned} \tag{2}
$$

3 Numerical Results

To solve the system of equations (2), Dixit's baseline case considers the following parametrization: $k = 2$, $\rho = 0.025$, $l = 0$, $N = 100$. Foreign demand and foreign supply are given by

$$
\begin{aligned}
Q^d &= 360 - 160p \\
Q^S &= 110 + 40p
\end{aligned}
$$

Therefore, the net import demand function is

$$q = 250 - 200p$$

which finally gives the net import inverse demand function

$$p_n = 1.25 - \frac{n}{200}$$

$$w_n = 0.85 + \frac{n}{500}$$

Let sunk cost decrease with the number of firms settled in the foreign market. Decreasing sunk cost could be related to more transparent industries which could not only foster earlier penetration, but also incur a lower sunk cost. Therefore, we specify $K(n)$ by nesting Dixit's constant sunk cost ($k = 2$) for the first potential exporting firm, with $K'(n) < 0$.

In order to provide a general solution we consider three cases

$$\begin{aligned} K_1(n) &= k - \frac{n-1}{N-1} \\ K_2(n) &= \frac{k}{n^\alpha}, \qquad a > 0 \\ K_3(n) &= \left(k^2 - \gamma\,(n-1)\right)^{1/2}, \qquad \gamma > 0 \end{aligned}$$

where $K_1''(n) = 0$, which implies that the marginal contribution of the nth firm to the reduction of the sunk cost is constant and equal to $-(1/N-1)$. $K_2''(n) > 0$. Hence $K_2(n)$ is a convex function and the marginal contribution of the nth firm to the reduction of the sunk cost is decreasing with n and equal to $-\alpha k n^{-\alpha-1}$. $K_3''(n) < 0$ and $K_3(n)$ is a concave function and the marginal contribution of the nth firm to the reduction of the sunk cost is increasing with n and equal to $-\frac{\gamma}{2}\left(k^2 - \gamma\,(n-1)\right)^{-1/2}$.

We consider a 50 percent reduction in the sunk cost from the first entrant to the last entrant. Therefore, when no foreign (Japanese) firm is settled in the industry, the first firm in entering ($n = 1$) incurs a sunk cost of $k = K_1(1) = K_2(1) = K_3(1) = 2$. When the last foreign firm enters ($n = 100$) it incurs a sunk cost of 1. Therefore, to ensure that $K_1(100) = K_2(100) = K_2(100) = 1$, $\alpha = Log\,(2)\,/Log\,(100)$ and $\gamma = 3/99$. Given the value of the parameters, we can plot the different sunk cost functions in Figure 1.

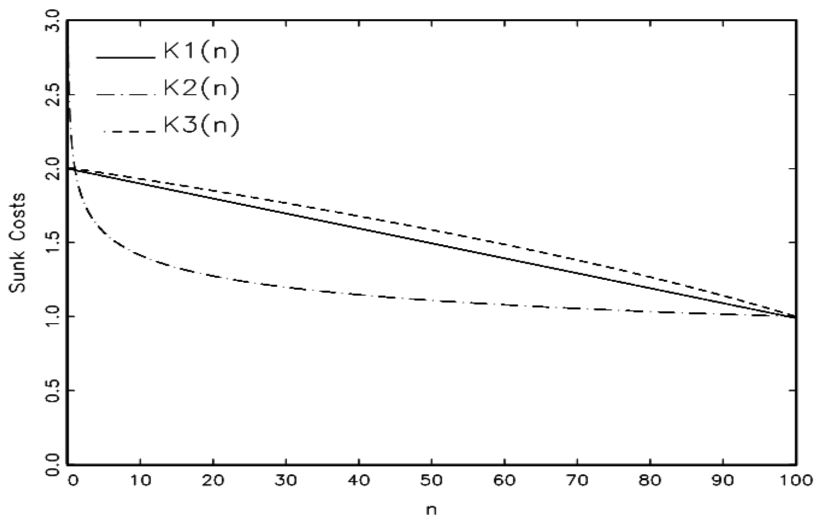

Figure 1. Sunk Cost Functions

It is well known that sunk costs act as a natural barrier. Moreover, we can see that the marginal contribution of the nth firm to the sunk cost can also act as a natural barrier. Therefore, departing from an initial sunk cost, $k = 2$, for the first firm, our numerical results show that when R reaches the value of one, the number of exporting firms is 27 in industries where the marginal contribution of the nth firm to the sunk cost is decreasing $(K_2(n))$. However, if the marginal contribution of the nth firm to the sunk cost is constant or increasing, the number of exporting firm is 21; and if there is no marginal contribution (Dixit's case), the number is 20. This result can shed light on policy implications. If information exchange is the via through which sunk cost can be reduced, and higher information gains come from previously entering firms, as in the case of $K_2(n)$, the role of the government in the earlier stages of penetration in a market could be crucial in providing information on the target market through official institutions and promoting the exchange of information among firms by fostering localization and urbanization economies. When R reaches the value of 1.76, regardless of the sunk cost function specified, all the firms have entered. However, with $K_2(n)$, all the firms except the first and the last have entered before and have incurred lower sunk costs.

Figure 2 shows the trigger exchange rates for the N firms. The solid lines denote the trigger exchange rates of Dixit (1989), while the dashed lines represent the trigger exchange rates with decreasing sunk costs. The exchange rates that cause foreign firms to enter through exporting is lower when the sunk costs are decreasing. As we pointed out above, the lower the sunk cost, the earlier the entry. However, the exchange rates that cause foreign firms to exit is higher. The cheaper it is to enter, the easier it is to abandon. Therefore, hysteresis is lower when sunk costs are decreasing.

Nevertheless, in all three cases of decreasing sunk costs with respect to the Dixit case (1989), the entry exchange rates diminish more than the rise in the exit exchange rates in both absolute and relative terms. This suggests that decreasing sunk costs have a greater impact on the option of activating than on the option of abandoning.

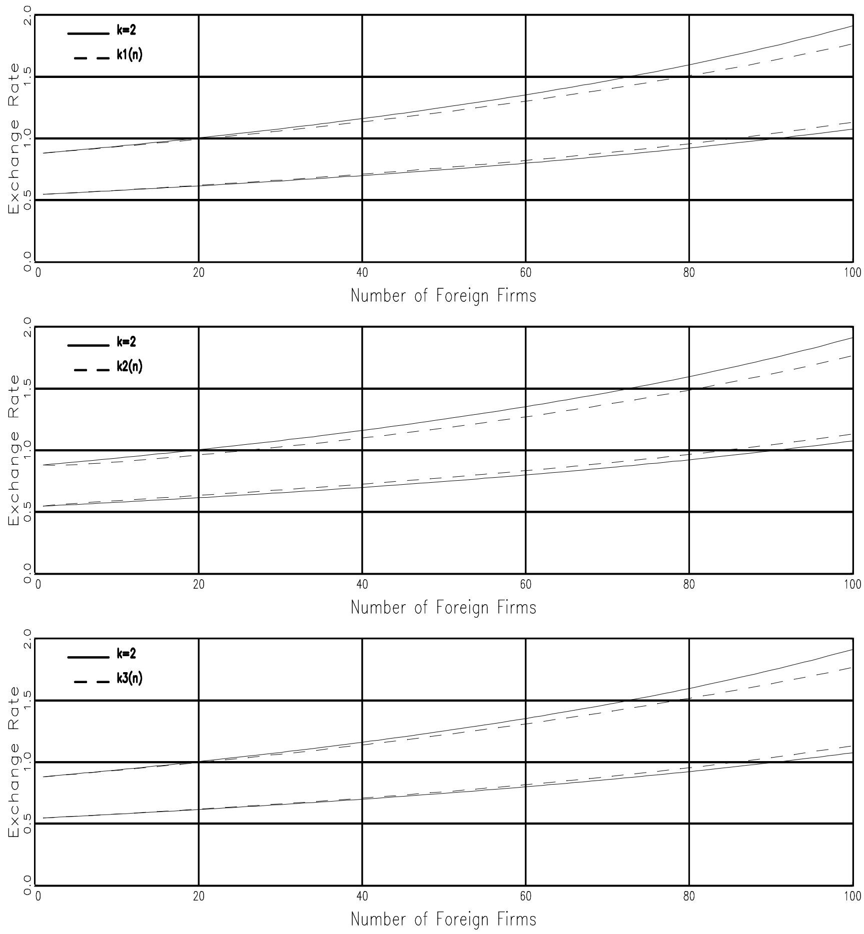

Figure 2. Trigger Exchange Rate for Entry and Exit.

When half of the firms have already settled in the industry ($n = 50$), the exchange rate that induces one firm to enter is 68 percent higher than the rate that induces one firm to exit with constant sunk cost. With lineal decreasing sunk cost it is 59 percent higher, with convex decreasing sunk cost it is 52 percent, and with concave decreasing sunk cost it is 61 percent. Equilibrium with 50 firms in the case of $k = 2$ is compatible with any exchange rate in the interval (0.75, 1.25); with $K_1(50)$, (0.76,1.21); with $K_2(50)$, (0.78, 1.18) and with $K_3(50)$, (0.76, 1.22). Therefore, the lower the sunk cost, the narrower the inaction bands, that is, the lower the hysteresis. Looking at this another way, an exchange rate equal to 1 is compatible with any number of firms in intervals: (20,90) with $k = 2$, (21,85) with $K_1(n)$ and $K_3(n)$, and (27,84) with $K_2(n)$. The lower the sunk costs, the narrower the range of the number firms in equilibrium.

Figure 3 shows the measure of hysteresis (I_n/D_n). As n increases from 1 to 100, the ratio I_n/D_n increases from 1.61 to 1.78 in the case of constant sunk cost ($k = 2$), while the ratio decreases from 1.61 to 1.56 with decreasing sunk cost. In the case of constant sunk cost, the growth of the entry trigger exchange is higher than the growth of the exit trigger exchange rate. With decreasing sunk costs $K_1(n)$ and $K_3(n)$, it is completely inverse. Therefore, unlike Dixit's case in which hysteresis increases as long as the firms enter, with decreasing sunk cost, $K_1(n)$ and $K_3(n)$, hysteresis is not only lower as we pointed out above, but is also decreasing with the number of firms in the market. The case of the decreasing sunk cost $K_2(n)$ is very striking. Notice that for the first third of entering firms (exactly $n < 34$), the growth of the entry trigger exchange rate is lower than the growth of the exit trigger exchange rate, thus giving rise to a decreasing ratio. However, with n$\geq$ 34, the growth of the entry trigger exchange rate is higher than that of the exit trigger exchange rate and hence an increasing ratio.

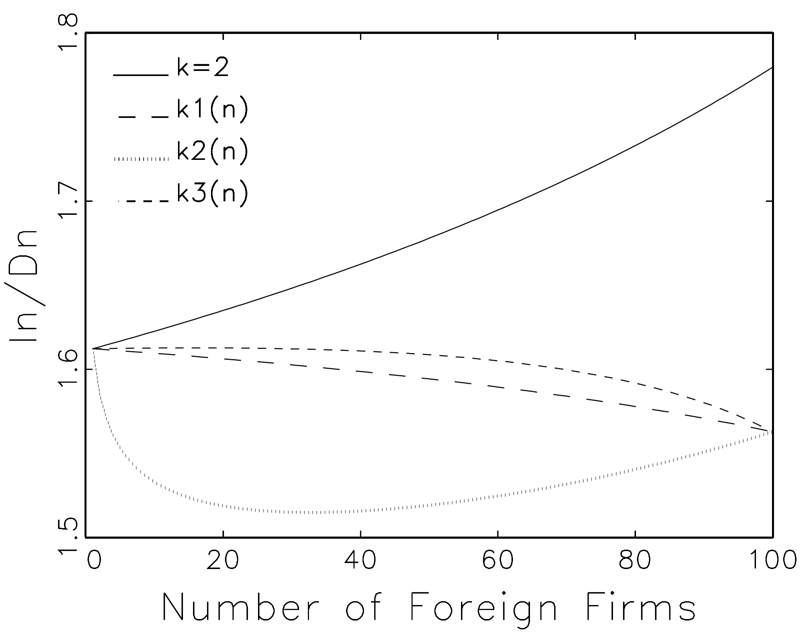

Figure 3. I_n/D_n Ratios across Different Sunk Cost Functions.

Starting from a normal initial state with $R = 1$ and $n = 50$, let us suppose that R rises by 40 percent. Table 1 shows the results. In the Dixit case, n rises to 64, that is, the importation share increases from the normal level of 25 percent to 30.3 percent. With decreasing sunk cost, n rises more and so the import share. Nevertheless, it is quite a small effect of a very large overvaluation. In the most sensitive case, $K_2(n)$, n rises to 72 and the import share increases to 33.1 percent and 32.1 percent for $K_1(n)$ and $K_3(n)$, respectively.

Let us also examine the response of the system to changes in the parameters around the central values after a 40 percent overvaluation. The first line of the Table 1 summarizes the results for the central case stated above. In the first variation, k doubles from 2 to 4, while w_n is offsetting to keep the normal full cost at the median equal to 1. With regard to the central case, the number of firms is reduced to 57 in the constant sunk case, that is, seven firms exit. In the case of the decreasing sunk cost, the number of exiting firms is also lower. Specifically, four firms exit with $K_1(n)$, five firms exit with $K_3(n)$ and only three firms exit with $K_2(n)$. In the second variation (σ halved) a large variation occurs in the number of entering firms: 10 in the case of constant sunk cost, 11 with $K_1(n)$ and $K_3(n)$ and 9 with $K_2(n)$. However, it is striking that the variations across different sunk cost functions are not so remarkable. Nevertheless, when wage is constant (w_n horizontal) and the range price is halved, a large variation is found with respect to the central case and between the constant sunk cost and decreasing sunk cost. In all three cases, the variation in the entering firms with decreasing sunk cost almost double the case of the constant sunk cost.

It is especially interesting to note the variation that produces changes in μ. Notice that the import share is very sensitive to a variation of $\pm 2\%$ in μ and more sensitive when $\mu = -0.02$ and when the sunk costs are higher. In the case of the constant sunk cost (the highest sunk cost), the import share is reduced by 5.7 points, while the reduction is 5.1 with $K_3(n)$ (the second highest sunk cost), 4.8 with $K_1(n)$ and 3.5 with $K_2(n)$. When $\mu = 0.02$, the variation in the import share shows a 4.8 point increase with k, 3.6 with $K_1(n)$ and $K_3(n)$ and 2.9 with $K_2(n)$. Therefore, considering an expected depreciation/appreciation rate of the same magnitude of the exchange rate, we find that the import share is more sensitive to exchange rate appreciations than to exchange rate depreciations. Moreover, under exchange rate appreciations, the larger the sunk cost, the larger the reduction in the import share. Under exchange rate depreciation, the larger the sunk cost, the larger the increase in the import share. Therefore, the larger the sunk cost, the larger the variation in import share due to exchange rate movements. Incidentally, it can be observed that higher positive values of μ reduce the differences across industry shares with a different sunk cost structure. This result suggests that in countries whose exchange rate suffers from exchange rate depreciation in the long term, the positive growth of the exchange rate hides the positive effects of decreasing sunk costs. Notice in Table 1 that with any sunk cost structure, with $\mu = 0.02$ and the central values of the parameters, the import share is about 35-36 percent. However, in countries whose exchange rate suffers from exchange rate appreciation in the long term, the negative growth of the exchange rate can be dampened by the positive effects of decreasing sunk costs.

Table 2 merely deserves comments. It shows that the exchange rate to start exiting and to restore $n = 50$ is higher with decreasing sunk costs. Therefore, import penetration is more "easy-come-easy-go" with decreasing sunk costs and even more so when these costs are lower.

4 Conclusion

In this article we have extended the Dixit model (*QJE*, 1989) to allow for a decrease in the sunk cost of entering a foreign market by exporting as long as the domestic firms establish in the target market. The numerical results show that, as regards the Dixit case, decreasing sunk costs have a greater impact on the option of activating than on the option of abandoning. For any given exchange rate, the lower the sunk cost, the narrower the range of number of firms in equilibrium in the industry. Moreover, hysteresis is lower with decreasing sunk cost as expected. The contribution of the article to this question is that unlike the Dixit case where hysteresis increases as long as new firms enter the market, hysteresis decreases with number of firms in industries where the marginal contribution to the reduction of the sunk cost of the new adding firm is constant or increasing. However, in industries with decreasing marginal contribution to the reduction of the sunk cost of the new adding firm, mixed results are found. For the first entering firms, the hysteresis decreases, while it increases for the rest of the firms. Given an expected depreciation/appreciation rate of the same magnitude in absolute value, import share is more sensitive under an appreciation environment. Moreover, the larger the sunk cost, the larger the variation in import share due to exchange rate movements. Finally, our results suggest that in countries whose exchange rate suffers from exchange rate depreciation in the long term, the positive growth of the exchange rate will hide the positive effects of decreasing sunk costs. In contrast, in countries whose exchange rate suffers from exchange rate appreciation in the long term, the negative growth of the exchange rate can be dampened by the positive effects of decreasing sunk costs.

Table 1: Effects of 40 Percent Overvaluation on the entry of firms

	Entry							
	k		K_1		K_2		K_3	
	n	%	n	%	n	%	n	%
Central Case	64	30.3	69	32.1	72	33.1	69	32.1
k doubled	57	27.7	65	30.7	69	32.1	64	30.3
σ halved	74	33.7	80	35.7	81	36.0	79	35.4
w_n horizontal	68	31.7	77	34.7	79	35.4	76	34.4
p_n range halved	72	33.1	84	37.0	86	37.6	83	36.7
$\mu = -2\%$	49	24.6	56	27.3	62	29.6	55	27.0
$\mu = +2\%$	78	35.1	80	35.7	81	36.0	80	35.7

Table 2: Effects of 40 Percent Overvaluation on the Rate Needed to Start Exit and Restore n=50

	Rate needed to							
	Start Exit				Restore n=50			
	k	K_1	K_2	K_3	k	K_1	K_2	K_3
Central Case	0.816	0.878	0.910	0.874	0.750	0.767	0.783	0.764
k doubled	0.700	0.767	0.805	0.758	0.670	0.690	0.707	0.688
σ halved	0.975	1.042	1.065	1.032	0.822	0.837	0.847	0.836
w_n horizontal	0.815	0.884	0.904	0.875	0.750	0.766	0.781	0.763
p_n range halved	0.820	0.902	0.918	0.895	0.750	0.765	0.781	0.763
$\mu = -2\%$	0.801	0.849	0.893	0.841	0.801	0.819	0.826	0.817
$\mu = +2\%$	0.822	0.892	0.913	0.886	0.680	0.713	0.737	0.708

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