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Balance-constrained growth rates: generalizing Thirlwall's law*

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Abstract

In this paper, we show how the concept of balance of payments-constrained growth rate developed by Thirlwall (1979) can be generalized as the growth rate constrained by the balance between any two economic variables. In particular, we derive two new concepts: the government balance-constrained growth rate, and the private balance-constrained growth rate. Some extensions of the basic model are also provided.

JEL classification: E20, F43, H62, O40

Key words: Economic growth, Balance of payments, Government balance, Private balance, Thirlwall's law

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1. Introduction

A well-known approach to the relationship between the balance of payments and economic growth is due to Thirlwall (1979). According to this author, the balance of payments can act as a constraint on the rate of growth of output, since it puts a limit on the growth in the level of demand to which supply can adapt. An increase in domestic output, by increasing imports, can lead to a deficit in the balance of payments, which may require either a fall in demand or a real exchange rate depreciation (i.e., a worsening of the terms of trade) in order to ensure the sustainability of the external deficit. Hence, an unsustainable external deficit sooner or later requires a correction, which puts a brake on further output growth.

From here, and assuming that the real exchange rate stays relatively constant, the concept of balance of payments-constrained growth rate follows, as the ratio of the rate of growth of exports to the income elasticity of the demand for imports. Only when the actual growth rate is lower than the balance of payments-constrained rate a country would be able to experience a sustained growth, on allowing equilibrium in the balance of payments. This rule (also known as Thirlwall's law) is the dynamic analogue of the Harrod trade multiplier (Harrod, 1933), and implies that a country growing above this balance of payments-constrained growth rate will run an external deficit, which would harm its future growth prospects; conversely, a country growing below will run an external surplus. This concept, on the other hand, is equivalent to a result derived by Krugman (1989), who found that countries growing faster face a higher income elasticity for their exports than for their imports. Thirlwall's model has been extensively tested over time, and provides a useful approximation to the growth experiences of both advanced and emerging countries. A historical background to the approach, together with an overview of empirical studies, is given in Thirlwall (2011); a collection of papers on the subject is McCombie and Thirlwall (2004).

In this paper, we show that Thirlwall's result can be generalized as the growth rate constrained by the balance between any two economic variables. In particular, we will derive two new concepts: the government balance-constrained growth rate, and the private balance-constrained growth rate. Some extensions of the basic model will be also provided.

2. Balance-constrained growth rates

Let's begin by introducing some notation: for any variable *Z*, $\hat{Z}$ indicates its rate of growth, and ε_{ZH} its elasticity with respect to variable *H*. Assume we have two economic variables, *A* and *B*, where *B* depends on the level of income, *Y*; and that, as the economy grows, we wish to keep unchanged the balance between *A* and *B*. Then, we should have:

$$\frac{dA}{dt} = \frac{dB}{dt}$$

where *t* denotes time.

Multiplying and dividing each side of this equation by *A* and *B*, respectively, and assuming that initially *A* = *B*, we find:

$$\hat{A} = \varepsilon_{BY}\hat{Y}$$

and hence the *growth rate constrained by the balance between A and B* will be:

$$\hat{Y}_{AB} = \frac{\hat{A}}{\varepsilon_{BY}} \qquad (1)$$

When A and B represent exports, X, and imports in domestic currency, M/Q (where Q is the real exchange rate, measured as the price of domestic goods relative to foreign goods), we have Thirlwall's *balance of payments-constrained growth rate* as a particular case of equation (1):

$$\hat{Y}_{BP} = \frac{\hat{X}}{\varepsilon_{MY}}$$

so that a country growing above $\hat{Y}_{BP}$ will run an external deficit, and a country growing below $\hat{Y}_{BP}$ will run an external surplus.

Recalling the fundamental macroeconomic identity:

$$(S - I) \equiv (G - T) + \left(X - \frac{M}{Q}\right)$$

where S, I, G, and T denote, respectively, private savings, private investment, government expenditure, and government revenue, we can derive two other particular cases of (1):

a) When $A = G$ and $B = T$, we obtain the *government balance-constrained growth rate*, i.e., the economy's rate of growth that allows equilibrium between government expenditures and revenues:

$$\hat{Y}_{GB} = \frac{\hat{G}}{\varepsilon_{TY}}$$

so that a country growing above $\hat{Y}_{GB}$ will have a government surplus, and a country growing below $\hat{Y}_{GB}$ will have a government deficit.

b) When $A = I$ and $B = S$, we obtain the *private balance-constrained growth rate*, i.e., the economy's rate of growth that allows equilibrium between private investment and savings:

$$\hat{Y}_{PB} = \frac{\hat{I}}{\varepsilon_{SY}}$$

so that a country growing above $\hat{Y}_{PB}$ will have an excess of private savings over private investment, and a country growing below $\hat{Y}_{PB}$ will have an excess of private investment over private savings.

3. Extensions

The result found in equation (1) can be generalized in the following way. We have assumed so far that *A* was exogenous, and *B* was a function of *Y*. Assume now that *A* and *B* depend on a set of variables $a_1,\ldots,a_n$, and $b_1,\ldots,b_p$, respectively, as well as on a set of common determinants $c_1,\ldots,c_k$; for the sake of completeness, assume that *A* also depends on *Y*:

$$A = f\,(a_1,\ldots,a_n;\, c_1,\ldots,c_k;\, Y)$$

$$B = g\,(b_1,\ldots,b_p;\, c_1,\ldots,c_k;\, Y)$$

Then, making $\frac{dA}{dt} = \frac{dB}{dt}$ and assuming again that initially $A = B$, implies:

$$\sum_{i=1}^{n} \varepsilon_{Aa_i}\,\hat{a_i} + \sum_{i=1}^{k} \varepsilon_{Ac_i}\,\hat{c_i} + \varepsilon_{AY}\hat{Y} = \sum_{i=1}^{p} \varepsilon_{Bb_i}\,\hat{b_i} + \sum_{i=1}^{k} \varepsilon_{Bc_i}\,\hat{c_i} + \varepsilon_{BY}\hat{Y}$$

or:

$$\sum_{i=1}^{n} \varepsilon_{Aa_i}\,\hat{a_i} - \sum_{i=1}^{p} \varepsilon_{Bb_i}\,\hat{b_i} + \sum_{i=1}^{k}\left(\varepsilon_{Ac_i} - \varepsilon_{Bc_i}\right)\hat{c_i} = (\varepsilon_{BY} - \varepsilon_{AY})\hat{Y}$$

so that the growth rate constrained by the balance between *A* and *B* becomes:

$$\hat{Y}_{AB} = \frac{\sum_{i=1}^{n} \varepsilon_{Aa_i}\,\hat{a_i} - \sum_{i=1}^{p} \varepsilon_{Bb_i}\,\hat{b_i} + \sum_{i=1}^{k}\left(\varepsilon_{Ac_i} - \varepsilon_{Bc_i}\right)\hat{c_i}}{(\varepsilon_{BY} - \varepsilon_{AY})} \tag{2}$$

As an example, notice that, if $A = X$ and $B = M/Q$, and X depends on foreign income, Y^*, the balance of payments-constrained growth rate will be:

$$\hat{Y}_{BP} = \frac{\varepsilon_{XY^*}\hat{Y}^*}{\varepsilon_{MY}}$$

while if, in addition, both exports and imports depend on the real exchange rate, Q, the latter becomes:

$$\hat{Y}_{BP} = \frac{\varepsilon_{XY^*}\hat{Y}^* + \left(1 + \varepsilon_{XQ} - \varepsilon_{MQ}\right)\hat{Q}}{\varepsilon_{MY}}$$

being both expressions particular cases of equation (2).

Further, as a second extension, assume that in the starting situation there is no equilibrium between A and B, so that $A + F = B$. Notice that, in the cases we have analysed before, F would represent net capital inflows, when $A = X$ and $B = M/Q$; a net reduction in government debt, when $A = G$ and $B = T$; or a net loan to the rest of the economy, when $A = I$ and $B = S$. Now, finding the growth rate constrained by the balance between A and B requires

$$\frac{dA}{dt} + \frac{dF}{dt} = \frac{dB}{dt}$$

which, after dividing by B, leads to:

$$(1 - \varphi)\hat{A} + \varphi\hat{F} = \varepsilon_{BY}\hat{Y}$$

where $\varphi = \frac{F}{B}$; and the growth rate constrained by the balance between A and B is:

$$\hat{Y}_{AB} = \frac{(1 - \varphi)\hat{A} + \varphi\hat{F}}{\varepsilon_{BY}} \qquad (1')$$

Finally, if we consider the most general case, i.e., when $A = f\,(a_1,\ldots,a_n;\ c_1,\ldots,c_k;\ Y)$ and $B = g\,(b_1,\ldots,b_p;\ c_1,\ldots,c_k;\ Y)$, and make $\frac{dA}{dt} + \frac{dF}{dt} = \frac{dB}{dt}$, we find, after dividing by B:

$$(1-\varphi)\left[\sum_{i=1}^{n} \varepsilon_{Aa_i}\,\widehat{a_i} + \sum_{i=1}^{k} \varepsilon_{Ac_i}\,\widehat{c_i} + \varepsilon_{AY}\hat{Y}\right] + \varphi\hat{F} = \sum_{i=1}^{p} \varepsilon_{Bb_i}\,\widehat{b_i} + \sum_{i=1}^{k} \varepsilon_{Bc_i}\,\widehat{c_i} + \varepsilon_{BY}\hat{Y}$$

and, after rearranging:

$$\sum_{i=1}^{n}(1-\varphi)\varepsilon_{Aa_i}\,\widehat{a_i} - \sum_{i=1}^{p} \varepsilon_{Bb_i}\,\widehat{b_i} + \sum_{i=1}^{k}\big((1-\varphi)\varepsilon_{Ac_i} - \varepsilon_{Bc_i}\big)\,\widehat{c_i} + \varphi\hat{F}$$

$$= (\varepsilon_{BY} - (1-\varphi)\varepsilon_{AY})\hat{Y}$$

so that the growth rate constrained by the balance between A and B would be now:

$$\hat{Y}_{AB} = \frac{\sum_{i=1}^{n}(1-\varphi)\varepsilon_{Aa_i}\,\widehat{a_i} - \sum_{i=1}^{p} \varepsilon_{Bb_i}\,\widehat{b_i} + \sum_{i=1}^{k}\big((1-\varphi)\varepsilon_{Ac_i} - \varepsilon_{Bc_i}\big)\,\widehat{c_i} + \varphi\hat{F}}{(\varepsilon_{BY} - (1-\varphi)\varepsilon_{AY})} \qquad (2')$$

To conclude, recall that the expression for the balance of payments-constrained growth rate derived by Thirlwall and Hussain (1982):

$$\hat{Y}_{BP} = \frac{(1-\varphi)\,\varepsilon_{XY^*}\hat{Y}^* + \big(1 + (1-\varphi)\varepsilon_{XQ} - \varepsilon_{MQ}\big)\hat{Q} + \varphi\hat{F}}{\varepsilon_{MY}}$$

or, alternatively, when $\hat{Q} = 0$ and X is exogenous:

$$\hat{Y}_{BP} = \frac{(1-\varphi)\hat{X} + \varphi\hat{F}}{\varepsilon_{MY}}$$

are particular cases of equations (2’) and (1’), respectively.

4. Concluding remarks

In this paper, we have shown that the concept of balance of payments-constrained growth rate developed by Thirlwall (1979) can be generalized as the growth rate constrained by the balance between any two economic variables. In particular, we have derived two new concepts: the government balance-constrained growth rate, and the private balance-constrained growth rate. In addition, some extensions of the basic model have been also provided, namely, the cases in which the two variables depend on a set of possible determinants, and when there is no equilibrium between the two variables in the starting situation. To conclude, notice that the two new concepts might prove to be useful in empirical applications.

References

Harrod, R. (1933): *International Economics*, Cambridge University Press, Cambridge.

Krugman, P. (1989): “Differences in income elasticities and trends in real exchange rates”, *European Economic Review* 33, 1031-1054.

McCombie, J. S. L. and Thirlwall, A. P. (2004): *Essays on Balance of Payments Constrained Growth: Theory and Evidence*, Routledge, London.

Thirlwall, A. P. (1979): “The balance of payments constraint as an explanation of international growth rate differences”, *Banca Nazionale del Lavoro Quarterly Review* 128, 45-53.

Thirlwall, A. P. (2011): “Balance of payments constrained growth models: history and overview”, *PSL Quarterly Review* 64, 307-351.

Thirlwall, A. P. and Hussain, M. N. (1982): "The balance of payments constraint, capital flows and growth rate differences between developing countries", *Oxford Economic Papers* 34, 498-510.